Constraints on Supersymmetric Grand Unified Theories from Cosmology

Jonathan Rocher[1†], **Mairi Sakellariadou**[2⋆]

[1] Institut d'Astrophysique de Paris, $\mathcal{G}\mathbf{R}\varepsilon\mathbf{C}\mathcal{O}$, FRE 2435-CNRS, 98bis boulevard Arago, 75014 Paris, France

[2] Division of Astrophysics, Astronomy and Mechanics, Department of Physics, University of Athens, Panepistimiopolis, GR-15784 Zografos (Athens), Hellas and Institut d'Astrophysique de Paris, 98bis boulevard Arago, 75014 Paris, France

E-mail: †`rocher@iap.fr`, ⋆`msakel@cc.uoa.gr`, `sakella@iap.fr`

Abstract. Within the context of SUSY GUTs, cosmic strings are generically formed at the end of hybrid inflation. However, the WMAP CMB measurements strongly constrain the possible cosmic strings contribution to the angular power spectrum of anisotropies. We investigate the parameter space of SUSY hybrid (F- and D- term) inflation, to get the conditions under which theoretical predictions are in agreement with data. The predictions of F-term inflation are in agreement with data, only if the superpotential coupling κ is small. In particular, for SUSY SO(10), the upper bound is $\kappa \lesssim 7 \times 10^{-7}$. This fine tuning problem can be lifted if we employ the curvaton mechanism, in which case $\kappa \lesssim 8 \times 10^{-3}$; higher values are not allowed by the gravitino constraint. The constraint on κ is equivalent to a constraint on the SSB mass scale M, namely $M \lesssim 2 \times 10^{15}$ GeV.
The study of D-term inflation shows that the inflaton field is of the order of the Planck scale; one should therefore consider SUGRA. We find that the cosmic strings contribution to the CMB anisotropies is not constant, but it is strongly dependent on the gauge coupling g and on the superpotential coupling λ. We obtain $g \lesssim 2 \times 10^{-2}$ and $\lambda \lesssim 3 \times 10^{-5}$. SUGRA corrections induce also a lower limit for λ. Equivalently, the Fayet-Iliopoulos term ξ must satisfy $\sqrt{\xi} \lesssim 2 \times 10^{15}$ GeV. This constraint holds for all allowed values of g.

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1. Introduction

High energy physics and cosmology are two complementary areas with a rich and fruitful interface. They both enter the description of the physical processes during the early stages of our Universe. High energy physics leads to the notion of topological defects, which imply a number of cosmological consequences. Once we compare the theoretical predictions of models motivated by high energy physics against cosmological data, we induce constraints or, in other words, we fix the free parameters of the models. This is the philosophy of our study.

Even though the particle physics Standard Model (SM) has been tested to a very high precision, evidence of neutrino masses [1, 2, 3] proves that one should go beyond this model. An extension of the SM gauge group can be accomplished in the framework of Supersymmetry (SUSY). At present, SUSY is the only viable theory for solving the

gauge hierarchy problem. Moreover, in the supersymmetric standard model the gauge coupling constants of the strong, weak and electromagnetic interactions, with SUSY broken at the TeV-scale, meet at a single point $M_{\rm GUT} \simeq (2-3)\times 10^{16}$ GeV. These are called Supersymmetric Grand Unified Theories (SUSY GUTs). An acceptable SUSY GUT model should be consistent with the standard model as well as with cosmology. SUSY GUTs can provide the scalar field needed for inflation, they can explain the matter-antimatter asymmetry of the Universe, and they can provide a candidate for cold dark matter, known as the lightest superparticle.

Usually models of SUSY GUTs suffer from the appearance of undesirable stable topological defects, which are mainly monopoles, but also domain walls. Topological defects appear via the Kibble mechanism [4]. A common mechanism to get rid of the unwanted topological defects, is to introduce one or more inflationary stages. Inflation essentially consists of a phase of accelerated expansion which took place at a very high energy scale. In addition, inflation provides a natural explanation for the origin of the large scale structure and the associated temperature anisotropies in the Cosmic Microwave Background (CMB) radiation. On the other hand, one has to address the question of whether it is needed fine-tuning of the parameters of the inflationary model. This is indeed the case in some nonsupersymmetric versions of inflation and it leads to the naturalness issue. Even though SUSY models of hybrid inflation were for long believed to circumvent these fine-tuning issues, our study shows that this may not be the case, unless we invoke the curvaton mechanism for the origin of the initial density fluctuations.

The study we present here is the continuation of Ref. [5], where they were examined all possible Spontaneously Symmetry Breaking (SSB) schemes from a large gauge group down to the SM gauge group, in the context of SUSY GUTs. Assuming standard supersymmetric hybrid inflation, there were found all models which are consistent with high energy physics and cosmology. Namely, there were selected all models which can solve the GUT monopole problem, lead to baryogenesis after inflation and are consistent with proton lifetime measurements. That study led to the conclusion that in all acceptable SSB patterns, the formation of cosmic strings is unavoidable, and some times it is accompanied by the formation of embedded strings. The strings which form at the end of inflation have a mass which is proportional to the inflationary scale. Here, we find the inflationary scale, which coincides with the string mass scale. Since our analysis is within global supersymmetry, we examine whether the value of the inflaton is at least a few orders of magnitude smaller than the Planck scale. As we show, global supersymmetry is sufficient in the case of F-term inflation, while D-term inflation necessitates the supergravity framework.

We organise the rest of the paper as follows: In Section II, we discuss the theoretical framework of our study. We briefly review the choice of the gauge groups we consider and we state the results about SSB schemes allowed from particle physics and cosmology as well as the topological defects left after the last inflationary era. We briefly review the theory of CMB temperature fluctuations. We then discuss inflation within N=1 SUSY GUTs, first in the context of F-term and then in the context of D-term inflation. In Section III, we describe our analysis as well as our findings for the mass scale and the strings contribution to the CMB. We first discuss F-term and then that of D-term inflation. We show that F-term inflation can be addressed in the context of global supersymmetry. We then show that D-term inflation has to be studied within local supersymmetry and we give the scalar potential for D-term inflation, taking into account radiative and supergravity (SUGRA) corrections. We

round up with our conclusions in Section IV. In Appendix A, we list the allowed SSB schemes of large gauge groups down to the standard model, found in Ref. [5], which are allowed by particle physics and cosmology.

2. Theoretical Framework

2.1. Hybrid inflation within supersymmetric grand unified theories

Grand Unified Theories imply a sequence of phase transitions associated with the SSB of the GUT gauge group $\mathrm{G}_{\mathrm{GUT}}$ down to the standard model gauge group $\mathrm{G}_{\mathrm{SM}} = \mathrm{SU(3)}_{\mathrm{C}} \times \mathrm{SU(2)}_{\mathrm{L}} \times \mathrm{U(1)}_{\mathrm{Y}}$. The energy scale at which this sequence of SSBs starts is $M_{\mathrm{GUT}} \sim 3 \times 10^{16}$ GeV. As our Universe has undergone this series of phase transitions, various kinds of topological defects may have been left behind as the consequence of SSB schemes, via the Kibble mechanism [4]. Among the various kinds of stable topological defects, monopoles and domain walls are undesirable, since they lead to catastrophic cosmological implications, while textures do not have important cosmological consequences. To get rid of the unwanted topological defects one may employ the mechanism of cosmological inflation.

Considering supersymmetric grand unified theories is motivated by several constraints, coming from both particle physics and cosmology. It is indeed the only viable framework to solve the hierarchy problem. In addition, it allows for a unification of strong, weak and electromagnetic interactions at a sufficiently high scale to be compatible with proton lifetime measurements. From the point of view of cosmology, SUSY GUTs can provide a good candidate for dark matter, namely the lightest supersymmetric particle. Moreover, it offers various candidates for playing the rôle of the inflaton field and it can give naturally a flat direction for slow-roll inflation.

Supersymmetry can be formulated either as a global or as a local symmetry, in which case gravity is included and the theory is called supergravity. Global supersymmetry can be seen as a limit of supergravity and it is a good approximation provided the Vacuum Expectation Values (VEVs) of all relevant fields are much smaller than the Planck mass.

In a supersymmetric theory, the tree-level potential for a scalar field is the sum of an F-term and a D-term. These two terms have rather different properties and in all proposed hybrid inflationary models only one of the two terms dominates. In supersymmetric hybrid inflation, the superpotential couples an inflaton field to a pair of Higgs fields that are responsible for one symmetry breaking in the SSB scheme. This class of inflationary models is considered natural [6] within SUSY GUTs in the sense that the only extra field which is added, except the fields needed to build the GUT itself, is a singlet scalar field. This extra field is however likely to be anyway needed to build the GUT model, so that it constraints the Higgs fields to acquire a VEV. Moreover, it is not spoiled by radiative corrections and supergravity corrections can be kept small for F-term inflation. We would like to mention that generically, F-term inflation suffers from the so-called η-problem, because usually, the supergravity corrections induce contribution of order unity to the slow roll parameter $\eta \equiv M_{\mathrm{Pl}}^2 V''/V$. This effective mass term for the inflaton field would spoil the slow roll condition [7]. However, in the case of a minimal Kahler potential, which is what is considered hereafter, this problem is lifted by a cancelation of the problematic mass terms. This can however be seen as a fine tunning since a minimal kahler potential is not well motivated for example from the point of view of the string theory [7]. Finally

it has been stated that such inflationary models are successful in the sense that they do not require any fine tuning, but we show that this last point has to be revisited.

F-term inflation can occur naturally within the GUTs framework when for example, a GUT gauge group $G_{\rm GUT}$ is broken down to the SM at an energy $M_{\rm GUT}$ according to

$$G_{\rm GUT} \xrightarrow{M_{\rm GUT}} H_1 \xrightarrow[\Phi_+\Phi_-]{M_{\rm infl}} H_2 \longrightarrow G_{\rm SM} \ , \tag{1}$$

where Φ_+, Φ_- is a pair of GUT Higgs superfields in nontrivial complex conjugate representations, which lower the rank of the group by one unit when acquiring nonzero VEV. The inflationary phase takes place at the beginning of the symmetry breaking $H_1 \xrightarrow{M_{\rm infl}} H_2$.

F-term inflation is based on the globally supersymmetric renormalisable superpotential

$$W_{\rm infl}^{\rm F} = \kappa S(\Phi_+\Phi_- - M^2) \ , \tag{2}$$

where S is a GUT gauge singlet left handed superfield and Φ_+, Φ_- are as defined above, with κ and M two constants (M has dimensions of mass) which can both be taken positive with field redefinition. The chiral superfields S, Φ_+, Φ_- are taken to have canonical kinetic terms. The above superpotential is the most general superpotential consistent with an R-symmetry under which $W \rightarrow e^{i\beta}W$ $, \Phi_- \rightarrow e^{-i\beta}\Phi_-$ $, \Phi_+ \rightarrow e^{i\beta}\Phi_+$, and $S \rightarrow e^{i\beta}S$. We note that an R-symmetry can ensure that the rest of the renormalisable terms are either absent or irrelevant.

D-term inflation is derived from the superpotential

$$W_{\rm infl}^{\rm D} = \lambda S \Phi_+\Phi_- \ , \tag{3}$$

where S, Φ_-, Φ_+ are three chiral superfields and λ is the superpotential coupling. D-term inflation requires the existence of a nonzero Fayet-Iliopoulos term ξ, which can be added to the lagrangian only in the presence of a U(1) gauge symmetry, under which, the three chiral superfields have charges $Q_S = 0$, $Q_{\Phi_+} = +1$ and $Q_{\Phi_-} = -1$, respectively. Thus, D-term inflation requires a scheme, like

$$G_{\rm GUT} \times U(1) \xrightarrow{M_{\rm GUT}} H \times U(1) \xrightarrow[\Phi_+\Phi_-]{M_{\rm infl}} H \rightarrow G_{\rm SM} \ . \tag{4}$$

This extra U(1) gauge symmetry symmetry can be of a different origin [6]. In what follows, we consider a nonanomalous U(1) gauge symmetry. We note however that one could instead consider a situation realised in heterotic string theories, where there is an anomalous D-term arising from an anomalous $\rm U(1)_A$, which can contribute to the vacuum energy [8]. Clearly, the symmetry breaking at the end of the inflationnary phase implies that cosmic strings are always formed at the end of D-term hybrid inflation.

In the SSB schemes studied in Ref. [5], one can naturally incorporate an era of F-/D-term inflation. All SSB schemes from grand unified gauge groups $\rm G_{GUT}$ of rank at the most equal to 8 down to the standard model gauge group $\rm G_{SM} \times Z_2$ have been considered. Initially, the group $\rm G_{GUT}$ was chosen to be one of the following ones: SU(5), SO(10), SU(6), $\rm E_6$, SU(7), SO(14), SU(8), and SU(9). In addition, the choice of $\rm G_{GUT}$ was limited to simple gauge groups which contain $\rm G_{SM}$, have a complex representation, are anomaly free and take into account some of the major observationnal constraints of particle physics and cosmology. The above

Z_2 symmetry is a sub-group of the $\mathrm{U(1)_{B-L}}$ gauge symmetry which is contained in various gauge groups and it plays the rôle of R-parity. R-parity can only appear in grand unified gauge groups which contain $\mathrm{U(1)_{B-L}}$. There were considered as many possible embeddings of $\mathrm{G_{SM}}$ in $\mathrm{G_{GUT}}$ as one can find in the literature and it was examined whether defects are formed during the SSB patterns of $\mathrm{G_{GUT}}$ down to $\mathrm{G_{SM}}$, and of which kind they are. Assuming standard hybrid F-term inflation there were disregarded all SSB patterns where monopoles or domain walls are formed after the end of the last possible inflationary era. In addition, there were disregarded SSB schemes with broken R-parity since the proton would decay too rapidly as compared to Super-Kamiokande measurements. It was also required that the gauged $\mathrm{U(1)_{B-L}}$ symmetry, is broken at the end of inflation so that a non thermal leptogenesis can explain the baryon/antibaryon asymmetry in the Universe. SU(5), SU(6), SU(7) are thus not acceptable groups for particle physics, since SU(5) leads to the formation of stable monopoles, while minimal SU(6) and minimal SU(7) do not contain $\mathrm{U(1)_{B-L}}$.

The SSB schemes compatible with high energy physics and cosmology, as given in detail in Ref. [5], are listed in Appendix A.

It was concluded that within the framework of the analysis of Ref. [5], there are not any acceptable SSB schemes without cosmic strings after the last inflationary era. The analysis we present below can therefore be applied to all these models. We would like to note that even if we relax the requirement that the gauged $\mathrm{B-L}$ symmetry is broken at the end of inflation, the results of Ref. [5] remain unchanged.

Even if one also allows for patterns with broken R-parity at low energy, cosmic strings formation is still very generic. To consider these cases, one should however find a mechanism to protect proton lifetime from very dangerous dimension 4 operators. Strings formed during the SSB phase transitions leading from a large gauge group $\mathrm{G_{GUT}}$ down to the $\mathrm{G_{SM}} \times Z_2$ are of two types: topological strings, called cosmic strings, and embedded strings which are not topologically stable and in general they are not dynamically stable either.

Clearly, as we discussed earlier, in the case of D-term inflation, cosmic strings are always present at the end of the inflationary era.

2.2. Cosmic microwave background temperature anisotropies

The CMB temperature anisotropies provide a powerful test for theoretical models aiming at describing the early Universe. The characteristics of the CMB multipole moments, and more precisely the position and amplitude of the acoustic peaks, as well as the statistical properties of the CMB temperature anisotropies, can be used to discriminate among theoretical models, as well as to constrain the parameters space.

The spherical harmonic expansion of the cosmic microwave background temperature anisotropies, as a function of angular position, is given by

$$\frac{\delta T}{T}(\mathbf{n}) = \sum_{\ell m} a_{\ell m} \mathcal{W}_\ell Y_{\ell m}(\mathbf{n}) \tag{5}$$

with

$$a_{\ell m} = \int \mathrm{d}\Omega_{\mathbf{n}} \frac{\delta T}{T}(\mathbf{n}) Y^*_{\ell m}(\mathbf{n}) \ , \tag{6}$$

where $\mathcal{W}_\ell$ stands for the ℓ-dependent window function of the particular experiment.

The angular power spectrum of CMB anisotropies is expressed in terms of the dimensionless coefficients C_ℓ, which appear in the expansion of the angular correlation function in terms of the Legendre polynomials P_ℓ:

$$\left\langle 0\left|\frac{\delta T}{T}(\mathbf{n})\frac{\delta T}{T}(\mathbf{n}')\right|0\right\rangle\Big|_{(\mathbf{n}\cdot\mathbf{n}'=\cos\vartheta)} = \frac{1}{4\pi}\sum_\ell (2\ell+1)C_\ell P_\ell(\cos\vartheta)\mathcal{W}_\ell^2 \ . \quad (7)$$

It compares points in the sky separated by an angle ϑ. Here, the brackets denote spatial average, or expectation values if perturbations are quantised ‡. The value of C_ℓ is determined by fluctuations on angular scales of order π/ℓ. The angular power spectrum of anisotropies observed today is usually given by the power per logarithmic interval in ℓ, plotting $\ell(\ell+1)C_\ell$ versus ℓ. The coefficients C_ℓ are related to $a_{\ell m}$ by

$$C_\ell = \frac{\langle \sum_m |a_{\ell m}|^2\rangle}{2\ell+1} \ . \quad (8)$$

On large angular scales, the main contribution to the temperature anisotropies is given by the Sachs-Wolfe effect, implying

$$\frac{\delta T}{T}(\mathbf{n}) \simeq \frac{1}{3}\Phi[\eta_{\rm lss}, \mathbf{n}(\eta_0 - \eta_{\rm lss})], \quad (9)$$

where $\Phi(\eta,\mathbf{x})$ is the Bardeen potential, while η_0 and $\eta_{\rm lss}$ denote respectively the conformal times now and at the last scattering surface. Note that the previous expression is only valid for the standard cold dark matter model.

If we assume that the initial density perturbations are due to "freezing in" of quantum fluctuations of a scalar field during an inflationary period, then the quadrupole anisotropy reads

$$\left(\frac{\delta T}{T}\right)_{\rm Q-infl} = \left[\left(\frac{\delta T}{T}\right)^2_{\rm Q-scal} + \left(\frac{\delta T}{T}\right)^2_{\rm Q-tens}\right]^{1/2} , \quad (10)$$

where the quadrupole anisotropy due to the scalar Sachs-Wolfe effect is

$$\left(\frac{\delta T}{T}\right)_{\rm Q-scal} = \frac{1}{4\sqrt{45}\pi}\frac{V^{3/2}(\varphi_Q)}{M_{\rm Pl}^3 V'(\varphi_Q)} \ , \quad (11)$$

and the tensor quadrupole anisotropy is

$$\left(\frac{\delta T}{T}\right)_{\rm Q-tens} \sim \frac{0.77}{8\pi}\,\frac{V^{1/2}(\varphi_Q)}{M_{\rm Pl}^2}. \quad (12)$$

We note that V is the potential of the inflaton field φ, with $V' \equiv {\rm d}V(\varphi)/{\rm d}\varphi$, while $M_{\rm Pl}$ denotes the reduced Planck mass, $M_{\rm Pl} = (8\pi G)^{-1/2} \simeq 2.43\times 10^{18}$ GeV, and $\varphi_{\rm Q}$ is the value of the inflaton field when the comoving scale corresponding to the quadrupole anisotropy became bigger than the Hubble radius.

The number of e-foldings of inflation between the initial value of inflaton φ_i and the final value φ_f is given by

$$N(\varphi_i \to \varphi_f) = -8\pi G\int_{\varphi_i}^{\varphi_f}\frac{V(\varphi)}{V'(\varphi)}{\rm d}\varphi \ . \quad (13)$$

The primordial fluctuations could also be generated from the quantum fluctuations of a late-decaying scalar field other than the inflaton, known as the

‡ We emphasise that Eq. (7) holds only if the initial state for cosmological perturbations of quantum-mechanical origin is the vacuum [9, 10].

curvaton field ψ [11, 12, 13, 14], whose nonvanishing initial amplitude is denoted by $\psi_{\rm init}$. During inflation the curvaton potential is very flat and ψ acquires quantum fluctuations [12]

$$\delta\psi_{\rm init} = \frac{H_{\rm inf}}{2\pi} \ , \tag{14}$$

where $H_{\rm infl}$ denotes the expansion rate during inflation, which is a function of the inflaton field, and it is given by the Friedmann equation

$$H^2_{\rm infl}(\varphi) = \frac{8\pi G}{3} V(\varphi) \ . \tag{15}$$

We assumed that the effective curvaton mass is much smaller than $H_{\rm infl}$, since otherwise the quantum fluctuations of the curvaton field during inflation would be very small and the CMB power spectrum would remain the same as in the standard adiabatic case.

At the end of the inflationary era, $\delta\psi_{\rm init}$ generates an entropy fluctuation. At later times, the curvaton field first dominates the energy density of the Universe, and then (during the RDE) it decays and reheats the Universe. Since the primordial fluctuations of the curvaton field are converted to purely adiabatic density fluctuations, the effect of $\delta\psi_{\rm init}$ in the CMB angular power spectrum, which can be parametrised by the metric perturbation induced by the curvaton fluctuation, is given by [15]

$$\Psi_{\rm curv} = -\frac{4}{9} \frac{\delta\psi_{\rm init}}{\psi_{\rm init}} \ . \tag{16}$$

There is no correlation between the primordial fluctuations of the inflaton and curvaton fields.

As we have explicitely shown in Ref. [5], the end of the inflationary era is accompanied by strings formation §, which are cosmic strings (topological defects), sometimes accompagnied by embedded strings (not topologically stable and in general not dynamically stable either). Let us first calculate the contribution to the quadrupole temperature anisotropy coming from the cosmic strings network.

At this point, we would like to bring to the attention of the reader that since the calculation of the angular power spectrum induced by a cosmic strings network relies on heavy numerical simulations, this issue remains still open. More precisely, to obtain the power spectrum from numerical simulations with cosmic strings, one must take into account the "three-scale model" [20] of cosmic strings networks, the small-scale structure (wiggliness) of the strings, the microphysics of the strings network, as well as the expansion of the Universe. This is indeed a rather difficult task, and to our knowledge no currently available simulation leading to the $C_\ell^{\rm strings}$ includes all of them. Moreover, all simulations of cosmic strings are referred to Nambu-Goto strings, and this is probably the most unrealistic case ‖ At least, but not last, there

§ Some authors have proposed mechanisms which could avoid cosmic strings production at the end of hybrid inflation. This is for example realised by adding a nonrenormalisable term in the superpotential [16], or by adding an additional discrete symmetry [17], or by considering GUTs models based on nonsimple and more complicated groups [18]. Moreover, by introducing a new pair of charged superfields in the framework of an N=2 supersting version of D-term inflation the strings which form are nontopological [19]. This last model was shown [19] to satisfy the CMB constraints.

‖ The nature of cosmic strings formed in the context of our models within SUSY GUTs and their cosmological rôle is studied in Ref. [21]. One has to examine whether the fermion zero modes which are intrinsic in supersymmetric models of cosmic strings lead to the production of vortons, which may result to a cosmological problem [22]. The microphysics of cosmic string solutions to N=1 supersymmetric abelian Higgs models has been studied in Ref. [23].

is the issue of the appropriate initial conditions for the time evolution of the cosmic strings network. More precisely, we do not know of any numerical simulation leading to the $C_\ell^{\rm strings}$, where the initial configuration of the strings network was other than the one obtained by assigning at random values to a phase variable on a cubic lattice. One should probably study whether the strings network is in the low or high density regime ¶.

Nevertheless, in what follows we use recent results [26] based on Nambu-Goto local strings simulations in a Friedmann–Lemaître–Roberston–Walker spacetime and we assume

$$\left(\frac{\delta T}{T}\right)_{\rm cs} \sim (9-10)G\mu \quad \text{with} \quad \mu = 2\pi\langle\chi\rangle^2 \ , \tag{17}$$

where $\langle\chi\rangle$ is the Vacuum Expectation Value (VEV) of the Higgs field responsible for the formation of cosmic strings.

2.3. Inflation in supersymmetric grand unified theories

In what follows, we first discuss inflation where the F-term dominates (F-term inflation) and then we address inflation where the D-term dominates (D-term inflation).

2.3.1. F-term inflation F-term inflation is based on the globally supersymmetric renormalisable superpotential Eq. (2). The scalar potential V can be obtained from Eq. (2) and it reads

$$V(\phi_+,\phi_-,S) = |F_{\Phi_+}|^2 + |F_{\Phi_-}|^2 + |F_S|^2 + \frac{1}{2}\sum_a g_a^2 D_a^2 \ , \tag{18}$$

where the F-term is such that[+] $F_{\Phi_i} \equiv |\partial W/\partial\Phi_i|_{\theta=0}$, with $\Phi_i = \Phi_+, \Phi_-, S$, and

$$D_a = \bar{\phi}_i \, (T_a)^i{}_j \, \phi^j + \xi_a \ , \tag{19}$$

with a the label of the gauge group generators T_a, g_a the gauge coupling, and ξ_a the Fayet-Iliopoulos term. By definition, in the F-term inflation the real constant ξ_a is zero; it can only be nonzero if T_a generates a U(1) group.

In the case of F-term inflation, the potential V as a function of the complex scalar component of the respective chiral superfields Φ_+, Φ_-, S reads

$$V^{\rm F}(\phi_+,\phi_-,S) = \kappa^2|M^2-\phi_-\phi_+|^2 + \kappa^2|S|^2(|\phi_-|^2+|\phi_+|^2) + {\rm D-terms}. \tag{20}$$

Assuming that the F-terms give rise to the inflationary potential energy density, while the D-terms are flat along the inflationary trajectory, one may neglect them during inflation. The D-terms may play an important rôle in determining the trajectory and in stabilising the noninflaton fields.

The potential has two minima: one valley of local minima ($V = \kappa^2 M^4 \equiv V_0$), for S greater than its critical value $S_{\rm c} = M$ with $\phi_+ = \phi_- = 0$, and one global supersymmetric minimum ($V = 0$) at $S = 0$ and $\phi_+ = \phi_- = M$. Imposing chaotic

¶ When the energy density of the strings network is low, the dominant part of the strings is in the form of closed loops of the smallest allowed size. At a certain critical density the strings network undergoes a phase transition characterised by the formation of *infinite* strings [24, 25].

[+] The notation $|\partial W/\partial\Phi_i|_{\theta=0}$ means that one has to take the scalar component (with $\theta = \bar{\theta} = 0$ in the superspace) of the superfields once one differentiates with respect to the superfields Φ_i. This is the reason for which the potential V is a function of the scalar fields ϕ_i.

initial conditions, i.e. $S \gg S_c$, the fields quickly settle down the valley of local minima. In the slow roll inflationary valley ($\phi_+ = \phi_- = 0$, $|S| \gg M$), the ground state of the scalar potential is different from zero, meaning that SUSY is broken. In the tree level, along the inflationary valley the potential $V = V_0$ is constant and thus it is perfectly flat. A slope along the potential can be generated by including the one-loop radiative corrections which are small as compared to V_0. Thus, the scalar potential acquires a little tilt which helps the scalar field S to slowly roll down the valley of minima. The SSB of SUSY along the inflationary valley by the vacuum energy density $\kappa^2 M^4$ leads to a mass splitting in the superfields Φ_+, Φ_-. One gets [28] a Dirac fermion with a mass squared term $\kappa^2|S|^2$ and two complex scalars with mass squared terms $\kappa^2|S|^2 \pm \kappa^2 M^2$. This implies that there are one-loop radiative corrections to V along the inflationary valley, which can be calculated using* the Coleman-Weinberg expression [27]

$$\Delta V_{1-\mathrm{loop}} = \frac{1}{64\pi^2}\sum_i (-1)^{F_i} m_i^4 \ln\frac{m_i^2}{\Lambda^2} , \tag{21}$$

where the sum extends over all helicity states i, with fermion number F_i and mass squared m_i^2; Λ stands for a renormalisation scale. Thus, the effective potential reads [28, 29, 30, 31, 32]

$$\begin{aligned} V_{\mathrm{eff}}^{\mathrm{F}}(|S|) &= V_0 + [\Delta V(|S|)]_{1-\mathrm{loop}} \\ &= \kappa^2 M^4 \left\{ 1 + \frac{\kappa^2 \mathcal{N}}{32\pi^2}\left[2\ln\frac{|S|^2\kappa^2}{\Lambda^2} + (z+1)^2\ln(1+z^{-1}) + (z-1)^2\ln(1-z^{-1}) \right] \right\} , \end{aligned} \tag{22}$$

where

$$z = \frac{|S|^2}{M^2} \equiv x^2 , \tag{23}$$

and $\mathcal{N}$ stands for the dimensionality of the representations to which the fields ϕ_+, ϕ_- belong.

Employing the above found expression, Eq. (22), for the effective potential we calculate in the next section the inflaton and cosmic strings contributions to the CMB temperature anisotropies.

2.3.2. D-term inflation In the context of global supersymmetry, D-term inflation is derived from the superpotential Eq. (3). In the global supersymmetric limit, Eqs.(18), (3) lead to the following expression for the scalar potential

$$V^{\mathrm{D}}(\phi_+, \phi_-, S) = \lambda^2 \left[|S|^2(|\phi_+|^2 + |\phi_-|^2) + |\phi_+\phi_-|^2 \right] + \frac{g^2}{2}(|\phi_+|^2 - |\phi_-|^2 + \xi)^2 , \tag{24}$$

where g is the gauge coupling of the U(1) symmetry and ξ is a Fayet-Iliopoulos term, chosen to be positive.

The potential has two minima. There is one global minimum at zero, reached for $|S| = |\phi_+| = 0$ and $|\phi_-| = \sqrt{\xi}$. There is also one local minimum, found by minimising the potential for fixed values of S with respect to the other fields. This local minimum is equal to $V_0 = g^2\xi^2/2$, reached for $|\phi_+| = |\phi_-| = 0$, with $|S| > S_c \equiv g\sqrt{\xi}/\lambda$. As in the previous discussed case (F-term inflation), also here the SSB of supersymmetry in the inflationary valley introduces a splitting in the masses of the components of the

* This expression has been derived in the case of a Minkowski background. However, during inflation the background geometry is given by the De Sitter metric, and therefore, strictly speaking, one should not use the standard Coleman-Weinberg expression, but one should instead find the corresponding expression in a De Sitter background.

chiral superfields Φ_+ and Φ_-. As a result, we obtain two scalars with squared masses $m^2_\pm = \lambda^2|S|^2 \pm g^2\xi$ and a Dirac fermion with squared mass $\lambda^2|S|^2$.

For arbitrary large S the tree level value of the potential remains constant and equal to $V_0 = (g^2/2)\xi^2$, thus S plays naturally the rôle of an inflaton field. Assuming chaotic initial conditions $|S| \gg S_{\rm c}$ one can see the onset of inflation. Along the inflationary trajectory the F-term vanishes and the Universe is dominated by the D-term, which splits the masses in the Φ_+ and Φ_- superfields, resulting to the one-loop effective potential for the inflaton field.

The radiative corrections given by the Coleman-Weinberg expression Eq. (21) lead to the following effective potential for D-term inflation

$$\begin{aligned} V_{\rm eff}^{\rm D}(|S|) &= V_0 + [\Delta V(|S|)]_{1-\rm loop} \\ &= \frac{g^2\xi^2}{2}\left\{1+\frac{g^2}{16\pi^2}\left[2\ln\frac{|S|^2\lambda^2}{\Lambda^2}+(z+1)^2\ln(1+z^{-1})+(z-1)^2\ln(1-z^{-1})\right]\right\}, \end{aligned} \tag{25}$$

with

$$z = \frac{\lambda^2|S|^2}{g^2\xi} \equiv x^2 \ . \tag{26}$$

Employing the above found expression for the effective potential, Eq. (25), we calculate in the next section the inflaton and cosmic strings contributions to the CMB temperature anisotropies.

3. Energy scale of inflation and inflaton/cosmic strings contributions to the CMB

3.1. F-term inflation in global supersymmetry

Assuming $V \simeq \kappa^2 M^4$, while using the exact expression for the potential as given in Eq. (22) for calculating $V' \equiv {\rm d}V/{\rm d}S$, we obtain

$$V'(|S|) = \frac{2a}{|S|} z\, f(z) \ , \tag{27}$$

with

$$a = \frac{\kappa^4 M^4 \mathcal{N}}{16\pi^2} \ , \tag{28}$$

and

$$f(z) = (z+1)\ln(1+z^{-1}) + (z-1)\ln(1-z^{-1}) \ . \tag{29}$$

Equation (11) implies [31, 32]

$$\left(\frac{\delta T}{T}\right)_{\rm Q-scal} \sim \frac{1}{\sqrt{45}}\sqrt{\frac{N_{\rm Q}}{\mathcal{N}}}\,\frac{M^2}{M_{\rm Pl}^2}\, x_{\rm Q}^{-1} y_{\rm Q}^{-1} f^{-1}(x_{\rm Q}^2) \ , \tag{30}$$

and Eq. (13) leads to

$$N_Q = \frac{4\pi^2}{\kappa^2\mathcal{N}}\frac{M^2}{M_{\rm Pl}^2}\, y_{\rm Q}^2 \ , \tag{31}$$

with

$$y_{\rm Q}^2 = \int_1^{x_{\rm Q}^2} \frac{{\rm d}z}{z f(z)} \ , \tag{32}$$

and

$$x_{\rm Q} = \frac{|S_{\rm Q}|}{M} \ . \tag{33}$$

We remind to the reader that the index $_{\rm Q}$ denotes the scale responsible for the quadrupole anisotropy in the CMB.

The coupling κ is related to the mass scale M, through the relation

$$\frac{M}{M_{\rm Pl}} = \frac{\sqrt{N_{\rm Q}\mathcal{N}}\,\kappa}{2\,\pi\,y_{\rm Q}} \ . \tag{34}$$

Equation (12) implies [31, 32]

$$\left(\frac{\delta T}{T}\right)_{\rm Q-tens} \sim \frac{0.77}{8\pi}\kappa\frac{M^2}{M_{\rm Pl}^2} \ . \tag{35}$$

Employing Eq. (17) in the case of F-term inflation, we express the cosmic strings contribution as

$$\left(\frac{\delta T}{T}\right)_{\rm cs} \sim \frac{9}{4}\left(\frac{M}{M_{\rm Pl}}\right)^2 \ . \tag{36}$$

Therefore, the total quadrupole anisotropy

$$\left[\left(\frac{\delta T}{T}\right)_{\rm Q-tot}\right]^2 = \left[\left(\frac{\delta T}{T}\right)_{\rm scal}\right]^2 + \left[\left(\frac{\delta T}{T}\right)_{\rm tens}\right]^2 + \left[\left(\frac{\delta T}{T}\right)_{\rm cs}\right]^2 \tag{37}$$

is explicitely given by

$$\left(\frac{\delta T}{T}\right)_{\rm Q-tot} \sim \left\{y_{\rm Q}^{-4}\left(\frac{\kappa^2\mathcal{N}\,N_{\rm Q}}{32\pi^2}\right)^2\left[\frac{64N_{\rm Q}}{45\mathcal{N}}x_{\rm Q}^{-2}y_{\rm Q}^{-2}f^{-2}(x_Q^2)+\left(\frac{0.77\kappa}{\pi}\right)^2+324\right]\right\}^{1/2} , \tag{38}$$

where the $(\delta T/T)_{\rm Q}^{\rm tot}$ is normalised to the Cosmic Background Explore (COBE) data [33], namely $(\delta T/T)_{\rm Q}^{\rm COBE} \sim 6.3\times 10^{-6}$. For given values of $\kappa, N_{\rm Q}, \mathcal{N}$, the above equation can be solved numerically for $x_{\rm Q}$, and then employing Eqs. (32) and (34), one obtains $y_{\rm Q}$ and M. We assume $N_{\rm Q} = 60$ and we find the inflationary scale M which is proportional to the string mass scale, as a function of the superpotential coupling κ for three values of $\mathcal{N}$. We choose $\mathcal{N} = \mathbf{27}, \mathbf{126}, \mathbf{351}$, which correspond to realistic SSB schemes in SO(10) or $\rm E_6$ models. The results are shown in Fig. 1 below.

Clearly, the mass scale is of the order of 10^{15} GeV, and it grows very slowly with $\mathcal{N}$. Since it will be useful later to know approximately the evolution of the mass parameter M with respect to the coupling κ, we fit the curve for $\mathcal{N} = \mathbf{126}$, which is shown in Fig. 1, by

$$M(\kappa) \sim \begin{cases} 4.1\times 10^{15} + 5.3\times 10^{12}\ \ln(\kappa)\ \text{GeV} & \text{if} \quad \kappa\in[2\times 10^{-5}, 10^{-2}]\ , \\ 1.1\times 10^{16} + 6.3\times 10^{14}\ \ln(\kappa)\ \text{GeV} & \text{if} \quad \kappa\in[10^{-7}, 2\times 10^{-5}]\ . \end{cases} \tag{39}$$

The dimensionless coupling κ is subject to the gravitino constraint, as well as to the constraint imposed by the CMB temperature anisotropies. As we show below, the strongest constraint on κ is imposed by the measured CMB temperature anisotropies; the cosmic strings contribution to the power spectrum should be strongly suppressed in order not to contradict the data.

Firstly, we calculate the upper bound on the dimensionless superpotential coupling κ, as imposed by the gravitino constraint. After the inflationary era, our Universe enters the high entropy radiation dominated phase via the reheating process,

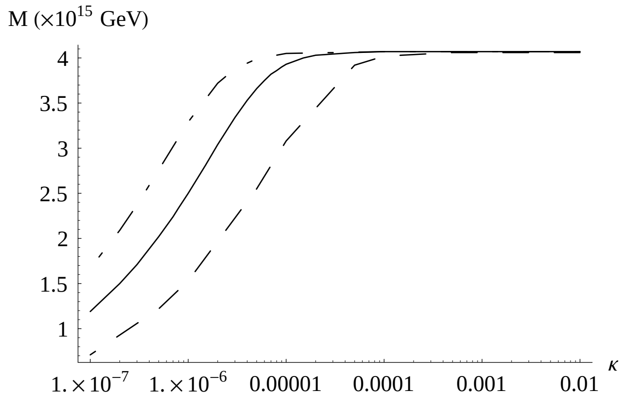


Figure 1. Evolution of the inflationary scale M in units of 10^{15} GeV as a function of the dimensionless coupling κ. The three curves correspond to $\mathcal{N} = \mathbf{27}$ (curve with broken line), $\mathcal{N} = \mathbf{126}$ (full line) and $\mathcal{N} = \mathbf{351}$ (curve with lines and dots).

during which the inflaton energy density decays perturbatively into normal particles. This process is characterised by the reheating temperature $T_{\rm RH}$. In order to have a successful reheating it is important not to create too many gravitinos.

Within the Minimal Supersymmetric Standard Model (MSSM), and assuming a see-saw mechanism to give rise to massive neutrinos, the reheating temperature is [35]

$$T_{\rm RH} \approx \frac{(8\pi)^{1/4}}{7} (\Gamma M_{\rm Pl})^{1/2} \ , \tag{40}$$

with Γ the decay width of the oscillating inflaton and the Higgs fields into right-handed neutrinos [35],

$$\Gamma = \frac{1}{8\pi} \left(\frac{M_i}{M}\right)^2 m_{\rm infl} \ , \tag{41}$$

$m_{\rm infl}$ the inflaton mass, and M_i the right handed neutrino mass eigenvalues. Equations (34), (40), (41) lead to

$$T_{\rm RH} \sim \frac{1}{12} \left(\frac{60}{N_{\rm Q}}\right)^{1/4} \left(\frac{1}{\mathcal{N}}\right)^{1/4} y_{\rm Q}^{1/2} M_i \ . \tag{42}$$

The reheating temperature must satisfy the gravitino constraint $T_{\rm RH} \leq 10^9$ GeV [36]. This constraint implies strong bounds on the M_i's, which satisfy the inequality $M_i < m_{\rm infl}/2$, where the inflaton mass is

$$m_{\rm infl} = \sqrt{2}\kappa M \ . \tag{43}$$

The strong bounds on M_i lead to quite small corresponding dimensionless couplings γ_i. The two heaviest neutrino are expected to have masses of the order of $M_3 \simeq 10^{15}$ GeV and $M_2 \simeq 2.5 \times 10^{12}$ GeV respectively [34]. This implies that for $y_{\rm Q}$ of the order of 1, M_i in Eq. (40) cannot be identified with the heaviest or the next to heaviest right handed neutrino, otherwise the reheating temperature would be higher than the upper limit imposed by the gravitino bound. Thus, M_i is M_1, with $M_1 \sim 6 \times 10^9$ GeV [34]. (This value is in agreement with the mass suggested in Ref. [35] for the mass of the right handed neutrino into which the inflaton disintegrates.)

Therefore, using Eqs. (40), (41) and (43), the gravitino constraint on the reheating temperature implies,

$$\frac{\sqrt{2}}{14\pi^{1/4}} M_1 \left[\frac{M_{\rm Pl}}{M(\kappa)}\right]^{1/2} \sqrt{\kappa} \leq 10^9 {\rm GeV} \ . \tag{44}$$

Using the fit of the function $M(\kappa)$ given in Eq. (39), it is possible to evaluate numerically $\sqrt{\kappa/M(\kappa)}$ and find an upper limit on the allowed values for the coupling of the superpotential,

$$\kappa \lesssim 8 \times 10^{-3} \ , \tag{45}$$

in agreement with the upper bound found in Ref. [37].

Secondly, we proceed with a strongest constraint on κ, imposed from the measurements on the CMB temperature anisotropies. The cosmic strings contribution to the quadrupole can be calculated from

$$\mathcal{A}_{\rm cs} \sim \left[\frac{\left(\frac{\delta T}{T}\right)_{\rm Q}^{\rm cs}}{\left(\frac{\delta T}{T}\right)_{\rm Q}^{\rm COBE}}\right]^2 \ , \tag{46}$$

and it is shown in Fig. 2.

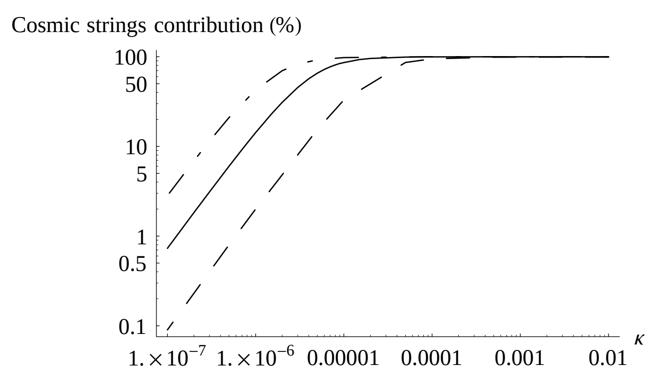


Figure 2. Evolution of the cosmic strings contribution to the quadrupole anisotropy as a function of the coupling of the superpotential, κ. The three curves correspond to $\mathcal{N} = \mathbf{27}$ (curve with broken line), $\mathcal{N} = \mathbf{126}$ (full line) and $\mathcal{N} = \mathbf{351}$ (curve with lines and dots).

As one can see, for small values of the coupling κ the cosmic strings contribution depends on the value of $\mathcal{N}$. Thus, we should find the value of $\mathcal{N}$ required for the allowed SSB patterns leading to $\mathrm{G_{SM}} \times Z_2$.

Within the framework of our study, the SSB schemes allowed from particle physics and cosmology are explicitely found in Ref. [5] and we recall them in Appendix A. The parameter $\mathcal{N}$ is the dimensionality of the Higgs fields representation that generate the SSB; these fields are coupled to the inflaton. As explained in Ref. [5], the inflationary era should take place after the last formation of monopoles and/or domain walls since these objects are incompatible with cosmology. Therefore, there are just very few choices for the SSB stage where inflation can be placed. The value of the parameter $\mathcal{N}$ depends on the GUT gauge group and the SSB scheme where inflation takes place.

For SO(10), under the requirement that R-parity is conserved down to low energies, $\mathcal{N} = \mathbf{126}$. For E_6, the Higgs representations can be $\mathcal{N} = \mathbf{27}$ or $\mathcal{N} = \mathbf{351}$. Our results depend slightly on the choice of $\mathcal{N}$ and in what follows, we focus mainly on $\mathcal{N} = \mathbf{126}$.

Comparing the results we obtained for the cosmic strings contribution to the CMB fluctuations as a function of the superpotential coupling κ, (see, Fig. 2) against the cosmic strings contribution allowed from the measurements, we can constrain κ. Already BOOMERanG [38], MAXIMA [39] and DASI [40] experiments imposed [41] an upper limit on $\mathcal{A}_{\rm cs}$, which is $\mathcal{A}_{\rm cs} \lesssim 18\%$. Clearly, the limit set by the Wilkinson Microwave Anisotropy Probe (WMAP) measurements [42] should be stronger. A recent Bayesian analysis in a three dimensional parameter space [43] have shown that a cosmic strings contribution to the primordial fluctuations $\mathcal{A}_{\rm cs}$ higher than 9% is excluded up to 99% confidence level.

We state below the constraint we found for the dimensionless parameter κ, as a function of $\mathcal{N}$,

$$\kappa \lesssim 7 \times 10^{-7} \times \frac{126}{\mathcal{N}} \ , \tag{47}$$

which holds for $\mathcal{N} \in \{\mathbf{1}, \mathbf{16}, \mathbf{27}, \mathbf{126}, \mathbf{351}\}$. This constraint on κ, Eq. (47), is in agreement with the one found in Ref. [44].

Both, the CMB lower bound and the gravitino upper bound on κ are summarised in Fig. 3.

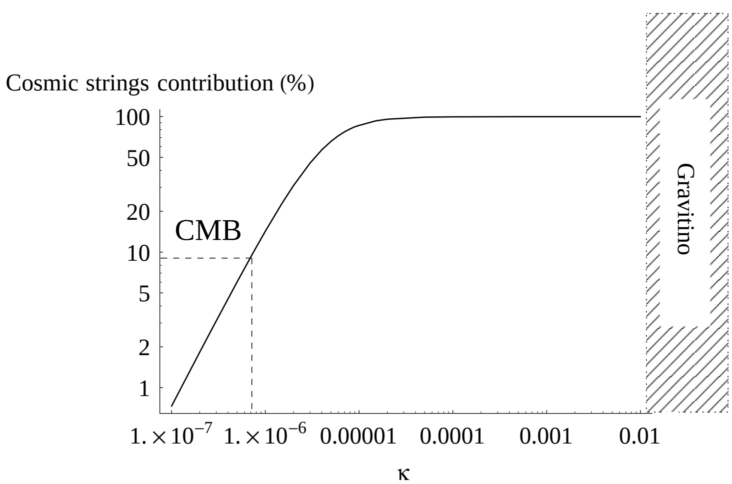


Figure 3. Constraints on the single parameter κ of the model. The gravitino constraint implies $\kappa \leq 8 \times 10^{-3}$. The allowed cosmic strings contribution to the CMB angular power spectrum implies $\kappa \lesssim 7 \times 10^{-7}$, for $\mathcal{N} = \mathbf{126}$.

The implications of this constraint on the single free parameter of the F-term inflationary model are important. Hybrid F-term inflation was known to have an appealing feature as compared to the most elegant inflationary scenario within nonsupersymmetric theories, i.e. chaotic inflation. Namely, it was believed that there was no need of fine tuning which will set a very small value for the coupling of the superpotential. This nice feature has been disappeared once we compare theoretical predictions against cosmological data. As we show below, one possible way out is to employ the curvaton mechanism. Another implication of this constraint, quite

important for cosmology, is the fact that this constraint on κ can be converted into a constraint on the mass parameter M. We would like to remind to the reader that this parameter controls the mass of the strings formed, as well as the inflationary scale. Using the CMB limit on the cosmic strings contribution, we can also obtain that, up to 99% of confidence level,

$$M \lesssim 2 \times 10^{15} \text{ GeV} . \tag{48}$$

As we show below, this constraint is robust since it holds even when the curvaton mechanism is involved, and for all values of $\mathcal{N}$.

In the curvaton scenario, the curvaton field is responsible for the generation of the primordial fluctuations. The curvaton is a scalar field, that is subdominant during the inflationary era as well as at the beginning of the radiation dominated era which follows the inflationary phase. The effective curvaton mass is assumed to be much smaller that the Hubble parameter during the inflationary phase. Within the framework of supersymmetry, which is the context of our study, one expects the existence of scalar fields. Thus, it is reasonable to expect that one of them could indeed play the rôle of the curvaton field. In addition, in the class of models we are considering, one may expect that it may exist a natural candidate for the curvaton field. As we have shown in Ref. [5], in many acceptable SSB schemes apart from the formation of topological cosmic strings we have the formation of embedded strings, which are topologically and dynamically unstable. If the decay product of embedded strings can give a scalar field before the onset of inflation, then such a scalar field could play the rôle of the curvaton field.

Assuming the existence of a curvaton field, there is an additional contribution to the temperature anisotropies. Thus,

$$\left[\left(\frac{\delta T}{T}\right)_{\rm tot}\right]^2 = \left[\left(\frac{\delta T}{T}\right)_{\rm infl}\right]^2 + \left[\left(\frac{\delta T}{T}\right)_{\rm cs}\right]^2 + \left[\left(\frac{\delta T}{T}\right)_{\rm curv}\right]^2 . \tag{49}$$

The inflaton contribution has a scalar and a tensor part, however since the tensor part in the supersymmetric hybrid inflation is always much smaller than the scalar one, we neglect it.

Since the primordial curvaton fluctuation is converted to purely adiabatic density fluctuations, the curvaton contribution in terms of the metric perturbation reads

$$\left(\frac{\delta T}{T}\right)_{\rm curv} \equiv \frac{\Psi_{\rm curv}}{3} , \tag{50}$$

and from Eq. (16) we have

$$\left(\frac{\delta T}{T}\right)_{\rm curv} = -\frac{4}{27}\frac{\delta\psi_{\rm init}}{\psi_{\rm init}} . \tag{51}$$

For $V \simeq \kappa^2 M^4$, Eqs. (14), (15), (34) and (51) imply

$$\left[\left(\frac{\delta T}{T}\right)_{\rm curv}\right]^2 = y_Q^{-4}\left(\frac{\kappa^2\mathcal{N}N_{\rm Q}}{32\pi^2}\right)^2\left[\left(\frac{16}{81\pi\sqrt{3}}\right)\kappa\left(\frac{M_{\rm Pl}}{\psi_{\rm init}}\right)\right]^2 , \tag{52}$$

We solve Eq. (49) using Eq. (52) and we obtain the different contributions of the three sources of anisotropies: inflaton, cosmic strings and curvaton. Their respective contributions as a function of κ, or $\psi_{\rm init}$, are shown in Fig. 4. for a fixed value of κ, the cosmic strings contribution decreases as $\psi_{\rm init}$ decreases, while the curvaton contribution becomes dominant. It is thus possible to use the WMAP constraint to

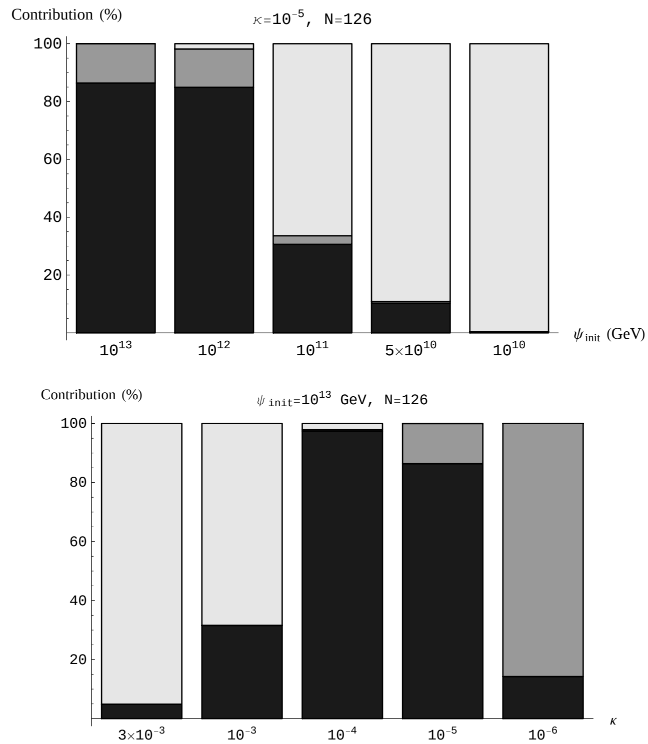


Figure 4. The cosmic strings (dark gray), curvaton (light gray) and inflaton (gray) contributions to the CMB temperature anisotropies as a function of the the initial value of the curvaton field $\psi_{\rm init}$, and the superpotential coupling κ, for $\mathcal{N} = \mathbf{126}$.

limit $\psi_{\rm init}$. The upper bound on $\psi_{\rm init}$ depends on the coupling κ. In addition, one has to impose the gravitino constraint. Therefore, a coupling bigger than 10^{-2} is excluded. More precisely,

$$\psi_{\rm init} \lesssim 5 \times 10^{13} \left(\frac{\kappa}{10^{-2}}\right) {\rm GeV} \ . \tag{53}$$

The above constraint is valid only if κ is in the range $[10^{-6},\ 1]$, while for smaller values of κ, the cosmic strings contribution is smaller than the WMAP limit for any value of the curvaton field (see, Fig. 4).

3.2. F-term inflation and supergravity corrections

We proceed with the supergravity corrections. The scalar potential in supergravity has the general form [45]

$$V = \frac{e^G}{M_{\rm Pl}^4}\left[G_i(G^{-1})^i_jG^j - 3\right] \quad \text{with} \quad G = \frac{K}{M_{\rm Pl}^2} + \ln\frac{|W|^2}{M_{\rm Pl}^6} \ , \tag{54}$$

where the Kähler potential $K(\phi,\phi^*)$ is a real function of the complex scalar fields ϕ_i, and their Hermitian conjugates $\phi_i{}^*$. The ϕ_i are scalar components of the chiral

superfields Φ_i. We adopted the following notations

$$G^i \equiv \frac{\partial G}{\partial \phi_i} \ , \quad G_j \equiv \frac{\partial G}{\partial \phi^{j*}} \ . \tag{55}$$

Assuming that the D-term is flat along the inflationary trajectory, one can ignore it during F-term inflation. Thus, we only consider the F-term in the above equation. If we expand $K = \sum_i |\phi_i|^2 + \cdots$, the corrections we get to the lowest order inflationary potential lead to the scalar potential

$$V = \sum_i \left|\frac{\partial W}{\partial \phi_i}\right|^2 \left(1 + \frac{1}{M_{\rm Pl}^2}\sum_i |\phi_i|^2 + \cdots\right) + \text{other terms} \ . \tag{56}$$

As it was shown in Ref. [7], assuming the minimal form for the Kähler potential K and minimising the scalar potential V with respect to ϕ_- and ϕ_+ for $|S| > M$, we obtain that the SUGRA correction to the scalar potential is

$$V_{\rm SUGRA} = \kappa^2 M^4 \left[\frac{1}{8}\frac{|S|^4}{M_{\rm Pl}^4} + \cdots\right] \ . \tag{57}$$

The effective scalar potential becomes

$$V_{\rm eff} = \kappa^2 M^4 \Bigg\{1 + \frac{\kappa^2 \mathcal{N}}{32\pi^2}\Big[2\ln\frac{|S|^2\kappa^2}{\Lambda^2} + (z+1)^2\ln(1+z^{-1}) + (z-1)^2\ln(1-z^{-1})\Big] + \frac{1}{8}\frac{|S|^4}{M_{\rm Pl}^4}\Bigg\} \ . \tag{58}$$

In the literature (e.g., see Ref. [28]), some authors consider only two terms for the scalar potential. Namely, they consider only the first term of the radiative corrections and the term coming from the SUGRA correction. However, this assumption holds only if $x_{\rm Q} \gg 1$. In our study we have found that the order of magnitude for $x_{\rm Q}$ is $\mathcal{O}(1)$ except for high values of the superpotential coupling κ (namely $\kappa \geq 10^{-2}$), but those are forbidden by the gravitino constraint. In conclusion, we find that all terms coming from the radiative corrections have to be taken into account.

Since $|S_{\rm Q}|/M_{\rm Pl} \lesssim 10^{-3}$, the potential and its first derivative is not significantly affected by the SUGRA corrections. Thus, including these corrections we do not expect to find any difference in the $\delta T/T$. This is indeed what one expects, since the value of the inflaton field, in the absence of SUGRA corrections, stays far below the Planck mass.

In conclusion, one can indeed neglect the supergravity corrections in the framework of F-term inflation.

3.3. D-term inflation in global supersymmetry

In what follows, we derive in the framework of global SUSY the cosmic strings contribution to the temperature anisotropies of the CMB. We use $V \simeq V_0 = g^2\xi^2/2$, while we employ Eq. (25) to derive the exact expression for $V' \equiv \partial V(|S|)/\partial |S|$, which reads

$$V'(|S|) = \frac{2b}{|S|} z\, f(z) \ , \tag{59}$$

where $f(z)$ is the same as in the F-term, Eq. (29), and

$$b = \frac{g^4 \xi^2}{16\pi^2}. \tag{60}$$

The next step is to express the number of e-foldings $N_{\rm Q}$. Employing the expressions written above for V and V' and using Eq. (13), we obtain

$$N_{\rm Q} = \frac{2\pi^2}{\lambda^2}\frac{\xi}{M_{\rm Pl}^2}\int_{z_{\rm end}}^{z_Q}\frac{{\rm d}z}{zf(z)} \ , \tag{61}$$

with

$$z_{\rm end} = \frac{\lambda^2|S_{\rm end}|^2}{g^2\xi} \ . \tag{62}$$

One has to evaluate the value of the inflaton field at the end of inflation. The inflationary era ends when one of the two following conditions is reached:

- In the global minimum of the potential, $\langle\phi_+\rangle$ or $\langle\phi_+\rangle$ is not zero anymore. This means that one of the two fields acquires a positive effective mass squared. Let us call $z_{\rm SB}$ the value of z for which this condition is realized.
- The slow roll conditions cease to be satisfied. Within supersymmetric hybrid inflation, the slow roll parameter ϵ is generically very small, and the slow roll condition becomes just $\eta \sim 1$. We denote by $z_{\rm SR}$ the value of z for which the slow roll condition ceases to be satisfied.

Thus,

$$z_{\rm end} = \max(z_{\rm SB}, z_{\rm SR}) \ . \tag{63}$$

Let us calculate the values of $z_{\rm SB}$ and $z_{\rm SR}$. Firstly, $z_{\rm SB}$ can be read from the quadratic part (for the fields ϕ_+ and ϕ_-) of the potential as it is given by Eq. (24). Namely, from the Lagrangian

$$\mathcal{L} = \lambda^2|S|^2(|\phi_+|^2 + |\phi_-|^2) + g^2\xi(|\phi_+|^2 - |\phi_-|^2) \ , \tag{64}$$

we get $z_{\rm SB} = 1$.

Secondly, the end of the slow roll phase of inflation is reached when the slow roll parameter η becomes of the order of unity,

$$\eta \equiv M_{\rm Pl}^2\left[\frac{V''(S)}{V(S)}\right] \sim 1 \ . \tag{65}$$

For our model,

$$\eta = \frac{\lambda^2}{4\pi^2}\left(\frac{M_{\rm Pl}^2}{\xi}\right)g(z) \ , \tag{66}$$

where

$$g(z) = (3z+1)\ln(1+z^{-1}) + (3z-1)\ln(1-z^{-1}) \ . \tag{67}$$

Equation (65) can be solved provided $\lambda \lesssim 4\times 10^{-3}$. This is a reasonable condition since higher values of λ would be forbidden by the gravitino constraint (see discussion in III.A). For $\lambda \lesssim 4\times 10^{-3}$, $\xi/\lambda^2 \gtrsim 10^{36}$ and as one can see from the shape of the function $g(z)$, the two solutions of Eq. (65) are very close to 1. This implies $z_{\rm SR} \sim 1$, and the number of e-foldings reads

$$N_{\rm Q} = \frac{2\pi^2}{\lambda^2}\frac{\xi}{M_{\rm Pl}^2}\,y_{\rm Q}^2 \ , \tag{68}$$

where $y_{\rm Q}$ is given by Eq. (32) with $x_{\rm Q} = \lambda|S_{\rm Q}|/(g\sqrt{\xi})$.

To measure the weight of the cosmic strings contribution, we suppose three sources for the quadrupole anisotropy of the CMB, namely

$$\left[\left(\frac{\delta T}{T}\right)_{\rm tot}\right]^2 = \left[\left(\frac{\delta T}{T}\right)_{\rm scal}\right]^2 + \left[\left(\frac{\delta T}{T}\right)_{\rm tens}\right]^2 + \left[\left(\frac{\delta T}{T}\right)_{\rm cs}\right]^2 , \quad (69)$$

and we calculate each term using Eqs. (11), (12), (17) respectively. In this last equation, the VEV of the Higgs field responsible of the cosmic strings formation is $\sqrt{\xi}$.
Thus, eliminating ξ with Eq. (68), we obtain

$$\left(\frac{\delta T}{T}\right)_{\rm Q-tot} \sim \left\{ y_{\rm Q}^{-4}\left(\frac{\lambda^2 N_{\rm Q}}{16\pi^2}\right)^2 \left[\frac{16N_{\rm Q}}{45} x_{\rm Q}^{-2} y_{\rm Q}^{-2} f^{-2}(x_Q^2) \right.\right. \\ \left.\left. + \left(\frac{0.77g}{\sqrt{2}\pi}\right)^2 + 324\right]\right\}^{1/2} , \quad (70)$$

where $(\delta T/T)_{\rm Q}^{\rm tot}$ is normalised to the COBE data, i.e., $(\delta T/T)_{\rm Q}^{\rm COBE} \sim 6.3\times 10^{-6}$. For given values of λ and $N_{\rm Q}$ we solve numerically Eq. (70) to get $x_{\rm Q}$.

Regarding D-term inflation we find that, as in the F-term case, the tensor contribution to the $\delta T/T$ is negligible. For $\lambda \lesssim 4\times 10^{-3}$, one can compute the mass scale $M_{\rm D} = \sqrt{\xi}$ as a function of λ. We find that ξ, which can be seen as a *mass parameter*, has a very similar shape as the one found for M in the case of F-term inflation (see, Fig. 1). Moreover, we obtain that the contribution of the cosmic strings to the quadrupole anisotropy is very similar to the F-term case. In addition, to be consistent with the CMB data, λ in D-term inflation should obey a similar limit to the one found for κ in F-term inflation, namely $\lambda \lesssim 3\times 10^{-5}$. These results are summarised in Fig. 5.

At this point we would like to bring to the attention of the reader that our findings regarding D-term inflation in SUSY, disagree with some results stated in previous studies♯ (see eg., Ref. [6]). The reason is that in our analysis, we take into account *all terms of the radiative corrections* and, as we have already shown in our discussion on F-term inflation, these terms have an essential impact on the determination of the cosmic strings contribution to the CMB.

Knowing $x_{\rm Q}$, one can compute from Eq. (26) the order of magnitude of the corresponding inflaton field $S_{\rm Q}$. Even though Eq. (70) is very similar to the one we had in the case of F-term hybrid inflation, i.e., Eq. (38), the conclusions for the value of the inflaton field as compared to the Planck mass are quite different. This is due to the dependence of $x_{\rm Q}$ on the coupling λ, which makes the inflaton field $S_{\rm Q}$ to be of the order of the Planck mass or higher.

The analysis we presented above implies that global supersymmetry is insufficient to study D-term inflation. It is clear that only in the context of supergravity one can treat correctly the issue of D-term inflation and its cosmological implications.

3.4. D-term inflation in supergravity

The case of D-term inflation has to be addressed within the framework of supergravity, since as we have shown in the previous subsection, the fields are not negligible as compared to the Planck mass.

♯ More precisely, we disagree with the statement that *in the context of D-term inflation cosmic strings contribute to the C_ℓ's up to the level of* 75%.

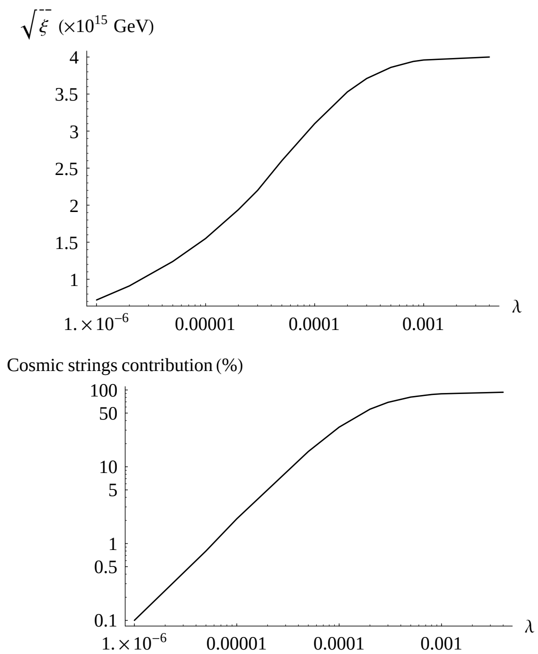


Figure 5. On the left, evolution of the mass scale $\sqrt{\xi}$ as a function of the coupling λ. On the right, evolution of the cosmic strings contribution to the quadrupole anisotropy as a function of the coupling of the superpotential, λ. These plots are derived in the framework of SUSY.

Inflation is still derived from the superpotential

$$W_{\rm infl}^{\rm D} = \lambda S \Phi_+ \Phi_- \ . \tag{71}$$

The F-term part of the scalar potential is given by Eq. (54). Considering also the D-term, the total scalar potential reads

$$V = \frac{e^G}{M_{\rm Pl}^4}\left[G_i (G^{-1})^i_j G^j - 3\right] + \frac{1}{2}[{\rm Re} f(\Phi_i)]^{-1} \sum_a g_a^2 D_a^2 \ , \tag{72}$$

where $f(\Phi_i)$ is the gauge kinetic function and

$$G = \frac{K}{M_{\rm Pl}^2} + \ln\frac{|W|^2}{M_{\rm Pl}^6} \ . \tag{73}$$

The Kähler potential $K(\phi_i, \phi_i^*)$ is a real function of the complex scalar fields ϕ_i, and their Hermitian conjugates ${\phi_i}^*$ where the ϕ_i are scalar components of the chiral superfields Φ_i. Upper (lower) indices (i,j) denote derivatives with respect to ϕ_i (${\phi^i}^*$). For the D-term part, ξ_a is the Fayet-Iliopoulos term, g_a the coupling of the $U(1)^a$ symmetry, which is generated by T_a and under which the chiral superfields S, Φ_+ and Φ_- have charges 0, +1 and −1 respectively. Finally,

$$D_a = \phi_i {(T_a)^i}_j K^j + \xi_a \ . \tag{74}$$

In what follows, we assume†† the minimal structure for $f(\Phi_i)$ (i.e., $f(\Phi_i)$=1) and take the minimal Kähler potential given by

$$K = |\phi_-|^2 + |\phi_+|^2 + |S|^2 \ . \tag{75}$$

Therefore, as it was found in [47, 48], the scalar potential reads

$$\begin{aligned} V^{\rm D}_{\rm SUGRA} &= \lambda^2 \exp\left(\frac{|\phi_-|^2+|\phi_+|^2+|S|^2}{M^2_{\rm Pl}}\right)\left[|\phi_+\phi_-|^2\left(1+\frac{|S|^4}{M^4_{\rm Pl}}\right)\right. \\ &\left. + \ |\phi_+S|^2\left(1+\frac{|\phi_-|^4}{M^4_{\rm Pl}}\right) + |\phi_-S|^2\left(1+\frac{|\phi_+|^4}{M^4_{\rm Pl}}\right) + 3\frac{|\phi_-\phi_+S|^2}{M^2_{\rm Pl}}\right] \\ &+ \frac{g^2}{2}\left(|\phi_+|^2 - |\phi_-|^2 + \xi\right)^2 \ . \end{aligned} \tag{76}$$

As in the case of global supersymmetry, for $S > S_{\rm c}$ the potential is minimised for $|\phi_+| = |\phi_-| = 0$ and therefore, at the tree level, the potential in the inflationary valley is constant, $V_0 = g^2\xi^2/2$. However, despite what is sometines claimed, this does not mean that the effective inflationary potential is identical to the one found in the case of global supersymmetry. One has to compute the radiative corrections, and must first calculate the effective masses of the components of the superfields Φ_+ and Φ_-. Extracting the quadratic terms from the potential, given by Eq. (76), the scalar components ϕ_+ and ϕ_- get squared masses

$$m^2_\pm = \lambda^2|S|^2 \exp\left(\frac{|S|^2}{M^2_{\rm Pl}}\right) \pm g^2\xi \ . \tag{77}$$

To calculate the masses of the fermionic components Ψ_+ and Ψ_-, one has to use the Lagrangian of the fermionic sector,

$$\mathcal{L}_{\rm fermion} = \frac{1}{2}e^{G/2}\left[-G^{ij} - G^iG^j + G^{ij}_k(G^{-1})^k_l G^l\right]\bar{\Psi}_{iL}\Psi_{jR} + {\rm h.c.}\ , \tag{78}$$

where Ψ_{iL} and Ψ_{jR} are left and right components of the Majorana spinors Ψ_+ and Ψ_-. Using Eq. (75), one gets a Dirac fermion with squared mass

$$m^2_{\rm f} = \lambda^2|S|^2\exp\left(\frac{|S|^2}{M^2_{\rm Pl}}\right) \ . \tag{79}$$

We calculate the radiative corrections $[\Delta V(|S|)]_{1-{\rm loop}}$ using the Coleman-Weinberg expression, Eq. (21). Thus, we compute the full effective scalar potential during D-term inflation, within the context of SUGRA

$$\begin{aligned} V^{\rm D-SUGRA}_{\rm eff} &= V_0 + [\Delta V(|S|)]_{1-{\rm loop}} \\ &= \frac{g^2\xi^2}{2}\left\{1 + \frac{g^2}{16\pi^2}\left[2\ln\frac{\lambda^2|S|^2}{\Lambda^2}\exp\left(\frac{|S|^2}{M^2_{\rm Pl}}\right)\right.\right. \\ &\left.\left. + (z+1)^2\ln(1+z^{-1}) + (z-1)^2\ln(1-z^{-1})\right]\right\} \end{aligned} \tag{80}$$

with

$$z = \frac{\lambda^2|S|^2}{g^2\xi}\exp\left(\frac{|S|^2}{M^2_{\rm Pl}}\right) \equiv x^2 \ . \tag{81}$$

†† This is the simplest supergravity model and in general the Kähler potential can be a more complicated function of the superfields. This is generally the case for supergravity theories derived from superstring theories.

The above potential is a generalisation of the SUSY D-term potential given by Eq. (25). The end of inflation is achieved when one of the two conditions is satisfied: (i) the symmetry is spontaneously broken, or (ii) the slow roll condition is violated. By definition of the variable z, the symmetry breaking condition is still

$$z_{\rm SB} = 1 \ . \tag{82}$$

For the slow roll condition, one has to compute the slow roll parameter η and then to solve $\eta \sim 1$. With the effective potential given by Eq. (80), one can compute its first derivative

$$V'(|S|) = \frac{2b}{|S|}\, z f(z) \left(1 + \frac{|S|^2}{M_{\rm Pl}^2}\right) \ . \tag{83}$$

Assuming $V \simeq V_0$ and using the exact expression for V', the η parameter reads

$$\eta(z) = \left(\frac{g^2}{16\pi^2}\frac{M_{\rm Pl}^2}{|S|^2}\right) z \left[g(z) + \frac{|S|^2}{M_{\rm Pl}^2} h_3(z) + \frac{|S|^4}{M_{\rm Pl}^4} h_4(z)\right] \ , \tag{84}$$

where $g(z)$ has been introduced for D-term inflation in Eq. (67), and $h_3(z), h_4(z)$ are given by

$$h_3(z) = (9z+5)\ln(1+z^{-1}) + (9z-5)\ln(1-z^{-1}) \ , \tag{85}$$

$$h_4(z) = (4z+2)\ln(1+z^{-1}) + (4z-2)\ln(1-z^{-1}) \ . \tag{86}$$

Supergravity corrections become essential for S at least one order of magnitude bigger that $M_{\rm Pl}$, and under this condition the solutions of $\eta(z) = 1$ are $z_{\rm SR}^{(1)} = 1^+$ and $z_{\rm SR}^{(2)} = 1^-$. Thus, z at the end of inflation, as defined by Eq. (63), is $z_{\rm end} = 1$.

The more complicated expression for $z(|S|)$ makes the calculation of the different contributions to the CMB power spectrum trickier. The cosmic strings contribution remains the same as the one computed in the case of SUSY D-term inflation, since it depends only on the scale $\sqrt{\xi}$. Concerning the inflaton contribution, we can use the dominant term of the potential $V \simeq V_0$ and the exact expression for the first derivative, Eq. (83), to write the number of e-foldings

$$N_{\rm Q} = \frac{2\pi^2}{g^2}\, \tilde{y}_{\rm Q}(x_{\rm Q}, \lambda, \xi, g) \ , \tag{87}$$

where

$$\tilde{y}_{\rm Q}(x_{\rm Q}, \lambda, \xi, g) = \int_1^{x_{\rm Q}^2} \frac{\mathrm{W}(z(g^2\xi)/(\lambda^2 M_{\rm Pl}^2))}{z^2 f(z)[1 + \mathrm{W}(z(g^2\xi)/(\lambda^2 M_{\rm Pl}^2))]^2}\, \mathrm{d}z \ . \tag{88}$$

We note that in the above definition of $\tilde{y}_{\rm Q}$, the function $\mathrm{W}(x)$ is the "W-Lambert function", i.e., the inverse of the function $F(x) = xe^x$. Setting $c \equiv (g^2\xi)/(\lambda^2 M_{\rm Pl}^2)$, the number of e-foldings $N_{\rm Q}$ becomes a function of only c and $x_{\rm Q}$, once g is fixed, and it is shown in Fig. 6. Imposing $N_{\rm Q} = 60$ allows us to find a numerical relation between c and $x_{\rm Q}$, as illustrated in Fig. 6.

Using this relation between c and $x_{\rm Q}$, it is possible to construct a function $x_{\rm Q}(\xi)$ and then express the three contributions (scalar, tensor and cosmic strings) to the CMB temperature anisotropies only as a function of the Fayet-Iliopoulos term ξ. Thus, we obtain that the total quadrupole temperature anisotropy reads

$$\left(\frac{\delta T}{T}\right)_{\rm Q}^{\rm tot} \sim \frac{\xi}{M_{\rm Pl}^2}\Bigg\{\frac{\pi^2}{90g^2} x_{\rm Q}^{-4} f^{-2}(x_{\rm Q}^2) \frac{\mathrm{W}(x_{\rm Q}^2(g^2\xi)(\lambda^2 M_{\rm Pl}^2))}{\Big[1 + \mathrm{W}(x_{\rm Q}^2(g^2\xi)(\lambda^2 M_{\rm Pl}^2))\Big]^2} + \left(\frac{0.77g}{8\sqrt{2}\pi}\right)^2 + \left(\frac{9}{4}\right)^2\Bigg\}^{1/2} \ , \tag{89}$$

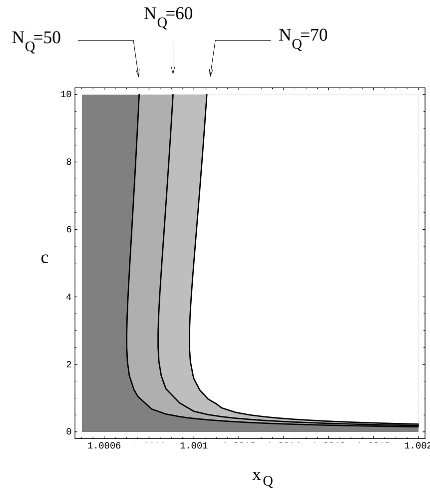


Figure 6. Iso-contours of $N_{\rm Q}$ for $N_{\rm Q} = 50, 60, 70$ in the plan $(x_{\rm Q}, c \equiv (g^2\xi)/(\lambda^2 M_{\rm Pl}^2))$. This graph is obtained for $g = 10^{-2}$.

where the only unknown is ξ, for given values of g and λ. We normalise the quadrupole anisotropy to COBE and we calculate numerically ξ, and thus $x_{\rm Q}$ using the function $x_{\rm Q}(\xi)$, as well as the various contributions as a function of the couplings λ and g.

Our results are listed below. Firstly, as previously, it is straightforward to see that the tensor contribution, even for $g = 1$, is completely negligible. Second, the inflaton field $S_{\rm Q}$ is of the order of $10 M_{\rm Pl}$ for the studied parameter space in λ and g whereas $M_{\rm D} = \sqrt{\xi}$ is still of the order of 2×10^{15} GeV. Concerning cosmic strings contribution to the CMB, we can see from Fig. 7 that it is not constant: it depends strongly on the value of the gauge coupling g and the superpotential coupling λ. For $g \gtrsim 1$, it is not possible that the D-term inflationary era lasts 60 e-foldings. Thus, a multiple stage inflation is necessary to solve the horizon problem. For $g \gtrsim 2 \times 10^{-2}$, the cosmic strings contribution is always greater than the WMAP limit, thus, it is ruled out.

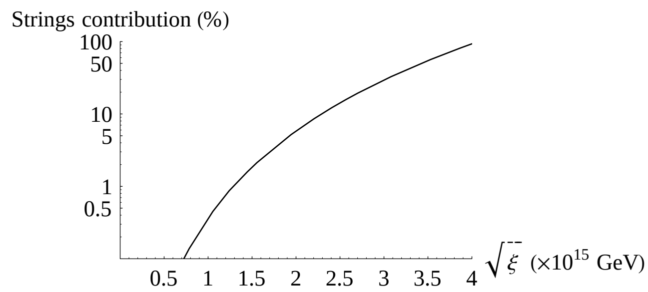
For high values of the superpotential coupling $\lambda \gtrsim 10^{-3}$, or for small values of the gauge coupling g (namely $g \lesssim 10^{-4}$), our findings are very similar to the ones obtained for D-term inflation within the SUSY context. This is expected since in these limits,

$$\mathrm{W}\left(\frac{g^2\xi}{\lambda^2 M_{\rm Pl}^2}\, z\right) = \frac{|S|^2}{M_{\rm Pl}^2} \ll 1 \ . \tag{90}$$

On the other hand, SUGRA corrections cannot be neglected for small values of λ, in which case the cosmic strings contribution rises again. It is thus possible to specify, for a given value of g, the allowed window for the superpotential coupling λ. As we stated earlier, our analysis shows that the allowed cosmic strings contribution to the CMB temperature anisotropies, as imposed by the WMAP measurements, leads to the constraint

$$g \lesssim 2 \times 10^{-2} \ . \tag{91}$$

Provided the above condition is satisfied, the CMB constraint sets an upper bound to



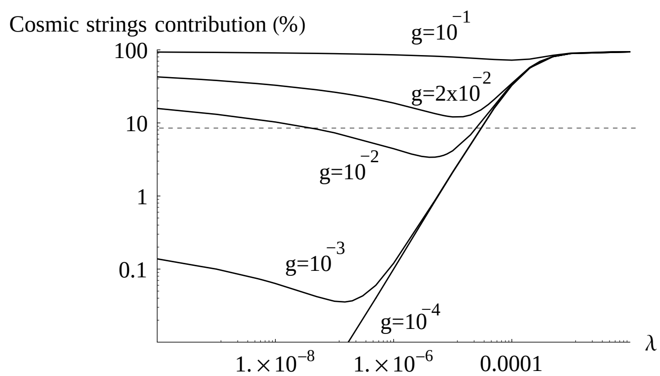


Figure 7. On the left, cosmic strings contribution as a function of the mass scale $\sqrt{\xi}$. This holds for all studied values of g. On the right, cosmic strings contribution to the CMB temperature anisotropies, as a function of the superpotential coupling λ, for different values of the gauge coupling g. The maximal contribution alowed by WMAP is represented by a dotted line. The plots are derived in the framework of SUGRA.

the allowed window for the superpotential coupling λ, namely

$$\lambda \lesssim 3 \times 10^{-5} \ , \tag{92}$$

up to a confidence level of 99%. This upper limit on λ is the same as our previous limit found in the framework of SUSY. Note that the upper bound we found for λ, Eq. (92), is in agreement with the one found in Ref.[44] in a slightly different model, where the gauge coupling g is related to the Yukawa coupling λ through the relation $\lambda = \sqrt{2}g$. Our constraint on λ, Eq. (92), is also in agreement with the finding $\lambda \lesssim \mathcal{O}(10^{-4}\text{–}10^{-5})$ of Ref. [46].

Supergravity corrections induce a lower bound for λ but for $g \lesssim 2 \times 10^{-3}$ it must be very small and it is therefore uninteresting, if one wants to avoid fine tuning of the parameters. However, this lower limit is interesting for g in the range of $[2 \times 10^{-3},\ 2 \times 10^{-2}]$. As an example, for $g = 10^{-2}$, the CMB constraint imposes

$$10^{-8} \lesssim \lambda \lesssim 3 \times 10^{-5} \ . \tag{93}$$

This allowed window for λ shrinks as g goes from $g = 10^{-2}$ to $g = 2 \times 10^{-2}$. This

is the most important contribution from supergravity. This fine tuning can be less severe if one invokes the curvaton mechanism [49].

As in the case of F-term inflation, it is also possible to convert this constraint into a constraint on the mass scale, which is given by the Fayet-Iliopoulos term ξ and we get

$$\sqrt{\xi} \lesssim 2 \times 10^{15} \text{ GeV} . \tag{94}$$

The above constraint on ξ is independent of the value of the gauge coupling g, provided Eq. (91) is satisfied.

We would like to bring to the attention of the reader that in the above study we have neglected the quantum gravitational effects, i.e., a contribution [50]

$$[\Delta V(|S|)]_{\mathrm{QG}} = C_1 \frac{\mathrm{d}^2 V}{\mathrm{d}S^2} \frac{V}{M_{\mathrm{Pl}}^2} + C_2 \frac{V^2}{M_{\mathrm{Pl}}^4} \tag{95}$$

(with C_1, C_2 numerical coefficients of the order of 1), to the effective potential, even though $S_{\mathrm{Q}} \sim \mathcal{O}(10 M_{\mathrm{Pl}})$. Our analysis is however still valid, since the effective potential given in Eq. (80) satisfies the conditions $V(|S|) \ll M_{\mathrm{Pl}}^4$ and $m_S^2 = \mathrm{d}^2 V/\mathrm{d}S^2 \ll M_{\mathrm{Pl}}^2$, and thus the quantum gravitational corrections $[\Delta V(|S|)]_{\mathrm{QG}}$ are very small as compared to the effective potential $V_{\mathrm{eff}}^{\mathrm{D-SUGRA}}$ [50].

4. Conclusions and discussion

In the context of SUSY GUTs, cosmic strings — occasionally accompanied by embedded strings — are the outcome of SSB schemes, compatible with high energy physics and cosmology. As it was explicitely shown in Ref. [5], strings are generically formed at the end of a supersymmetric hybrid inflationary era, which can be either F-term or D-term type, as we follow the patterns of SSBs from grand unified gauge groups $\mathrm{G}_{\mathrm{GUT}}$ down to the standard model gauge group $\mathrm{G}_{\mathrm{SM}} \times Z_2$. One should keep in mind, that there are mechanisms to avoid cosmic strings formation, which are not considered here since we focus on the most standard GUT and hybrid inflation models. The strings forming at the end of inflation have a mass which is proportional to the inflationary scale. However, current CMB temperature anisotropies data limit the contribution of cosmic strings to the angular power spectrum. The aim of this study is to use the data given by the realm of cosmology to constrain the parameters of supersymmetric hybrid inflationary models.

More precisely, in the framework of generic SUSY GUTs, we study the consequences of simple hybrid inflation models in the context of some realistic cosmological scenario. We then compare the observational predictions of these models in order to constrain the parameter's space. We perform our calculations within SUSY or SUGRA, whenever this is needed.

For F-term inflation, the symmetry breaking scale M associated with the inflaton mass scale (and the strings scale) is a slowly varying function of the superpotential coupling κ, and of the dimensionality $\mathcal{N}$ of the representations to which the scalar components of the chiral Higgs superfields belong. For SO(10), under the requirement that R-parity is conserved down to low energies, $\mathcal{N} = \mathbf{126}$. For E_6, the Higgs representations can be $\mathcal{N} = \mathbf{27}$ or $\mathcal{N} = \mathbf{351}$. The dependence of a measurable quantity (the cosmic strings contribution) on this discrete parameter $\mathcal{N}$ can provide an interesting tool to discriminate among different grand unified gauge groups.

The unique free parameter of the model, the dimensionless coupling κ, should obey two constraints. Namely, it should satisfy the gravitino constraint, and it should also obey to the CMB constraint. We found that the CMB constraint on the coupling κ is the strongest one: it imposes $\kappa \lesssim 7 \times 10^{-7} \times (126/\mathcal{N})$, for $\mathcal{N} \in \{\mathbf{1}, \mathbf{16}, \mathbf{27}, \mathbf{126}, \mathbf{351}\}$ whereas the gravitino constraint reads $\kappa \lesssim 8 \times 10^{-3}$.

The WMAP constraint on the superpotential coupling κ can be expressed into a constraint on the mass scale, namely $M \lesssim 2 \times 10^{15}$ GeV. The value of the inflaton field is of the same order of magnitude, and since it is below the Planck scale, it implies that global supersymmetry is sufficient for our analysis. Thus, it is not necessary to take into account supergravity corrections.

The implications of this finding are quite strong. Supersymmetric hybrid inflation was advertised to circumvent the naturalness issue appearing in the most elegant nonsupersymmetric inflationary model, i.e., chaotic inflation. Namely, chaotic inflation needed a very small coupling ($\lambda \sim 10^{-14}$). Cosmology taught us that supersymmetric hybrid inflation also suffers from the fine tuning issue when embedded within the class of cosmological models considered here.

In this study we also employed a mechanism where this problem can be lifted. This is done if we use the curvaton mechanism. In this case another scalar field, called the curvaton, could generate the initial density perturbations whereas the inflaton field is only responsible for the dynamics of the Universe. Within supersymmetric theories such scalar fields are expected to exist. In this case, the coupling κ is only constrained by the gravitino limit.

We performed the same analysis for D-term inflation. In this case we found that the value of the inflaton field is of the order of the Planck scale or higher. One should therefore consider local supersymmetry, taking into account all the one-loop radiative corrections. To our knowledge, this was never considered in previous studies. Our analysis gives a nonconstant contribution of cosmic strings to the CMB temperature anisotropies, which is strongly dependent on the gauge coupling g and the superpotential λ. We claim that the D-term inflationary model is still an open possibility, since it does not always imply that the cosmic strings contribution to the CMB is above the upper limit allowed by WMAP. To avoid contradiction with the data, the free parameters of the model are strongly constrained. More precisely, we found that the gauge coupling must satisfy the constraint $g \lesssim 2 \times 10^{-2}$, and the superpotential coupling must obey the condition $\lambda \lesssim 3 \times 10^{-5}$. This is the same limit as the one found in the SUSY framework. The supergravity corrections also give a lower limit on the value of this parameter. As an example, for $g = 10^{-2}$ we found $10^{-8} \lesssim \lambda \lesssim 10^{-8}$. This allowed window for λ shrinks as g goes from $g = 10^{-2}$ to $g = 2 \times 10^{-2}$. The conditions imposed by the CMB data on the couplings λ, g can be expressed as a single constraint on the Fayet-Iliopoulos term ξ, namely $\sqrt{\xi} \lesssim 2 \times 10^{15}$ GeV, which remains valid independently of g.

To conclude, we would like to emphasise that cosmic strings of the GUT scale are in agreement with observationnal data and can play a rôle in cosmology. CMB data do not rule out cosmic strings; they impose strong constraints on their possible contribution. One should use these constraints to test high energy physics such as supersymmetric grand unified theories. This is indeed the philosophy of our paper.

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5. Appendix

We list below the SSB schemes compatible with high energy physics and cosmology, as given in detail in Ref. [5]. Let us remind to the reader that every $\xrightarrow{n}$ represent an SSB during which there is formation of topological defects. Their nature is given by n: 1 for monopoles, 2 for topological cosmic strings, $2'$ for embedded strings, 3 for domain walls. Please note also that for e.g. $3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L}$ stands for the group $\mathrm{SU}(3)_{\rm C}\times \mathrm{SU}(2)_{\rm L}\times \mathrm{SU}(2)_{\rm R}\times \mathrm{U}(1)_{\rm B-L}$. We refer the reader to Ref. [5] for more details.

We first give the SSB schemes of SO(10) down to the $\mathrm{G_{SM}}\times Z_2$.

$$\mathrm{SO}(10)\left\{\begin{array}{lcclcl}
\xrightarrow{1} & 5\ 1_{\rm V} & \xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm Z}\ 1_{\rm V} & \xrightarrow{2} & \mathrm{G_{SM}}\ Z_2\\
\xrightarrow{1} & 4_{\rm C}\ 2_{\rm L}\ 2_{\rm R} & \longrightarrow & \text{Eq. (97)} & & \\
\xrightarrow{1,2} & 4_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ Z_2^{\rm C} & \longrightarrow & \text{Eq. (98)} & & \\
\xrightarrow{1,2} & 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ Z_2^{\rm C} & \longrightarrow & \cdots & & \\
\xrightarrow{1} & 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R} & \longrightarrow & \cdots & & \\
\xrightarrow{1,2} & 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L}\ Z_2^{\rm C} & \longrightarrow & \cdots & & \\
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L} & \longrightarrow & \cdots & & \\
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} & \xrightarrow{2} & \mathrm{G_{SM}}\ Z_2 & &
\end{array}\right. \tag{96}$$

where

$$4_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\left\{\begin{array}{ll}
\xrightarrow{1}\ 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L} & \left\{\begin{array}{l}\xrightarrow{1}\ 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L}\ \xrightarrow{2}\ \mathrm{G_{SM}}\ Z_2\\ \xrightarrow{2',2}\ \mathrm{G_{SM}}\ Z_2\end{array}\right.\\
\xrightarrow{1}\ 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R} & \left\{\begin{array}{l}\xrightarrow{1}\ 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L}\ \xrightarrow{2}\ \mathrm{G_{SM}}\ Z_2\\ \xrightarrow{2',2}\ \mathrm{G_{SM}}\ Z_2\end{array}\right.\\
\xrightarrow{1}\ 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} & \xrightarrow{2}\ \mathrm{G_{SM}}\ Z_2
\end{array}\right. \tag{97}$$

and

$$4_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ Z_2^{\rm C}\left\{\begin{array}{ll}
\xrightarrow{1}\ 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L}\ Z_2^{\rm C} & \left\{\begin{array}{l}\xrightarrow{3}\ 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L}\ \longrightarrow\ \cdots\\ \xrightarrow{1,3}\ 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L}\ \xrightarrow{2}\ \mathrm{G_{SM}}\ Z_2\end{array}\right.\\
\xrightarrow{1}\ 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ Z_2^{\rm C} & \left\{\begin{array}{l}\xrightarrow{3}\ 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ \longrightarrow\ \cdots\\ \xrightarrow{1,3}\ 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L}\ \xrightarrow{2}\ \mathrm{G_{SM}}\ Z_2\end{array}\right.\\
\xrightarrow{3}\ 4_{\rm C}\ 2_{\rm L}\ 2_{\rm R} & \longrightarrow\ \text{Eq. (97)}\\
\xrightarrow{1}\ 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R} & \longrightarrow\ \cdots\\
\xrightarrow{1,3}\ 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L} & \longrightarrow\ \cdots\\
\xrightarrow{1,3}\ 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} & \xrightarrow{2}\ \mathrm{G_{SM}}\ Z_2
\end{array}\right. \tag{98}$$

We then proceed with the list of the SSB schemes of E_6 down to the $\mathrm{G_{SM}} \times Z_2$. We first give the SSB patterns where E_6 is broken via $\mathrm{SO}(10) \times \mathrm{U}(1)$.

$$\mathrm{E}_6 \xrightarrow{1} 10\ 1_{\mathrm{V'}} \left\{ \begin{array}{llll} \xrightarrow{2} & 10 & \longrightarrow & \cdots \\ \xrightarrow{1} & 5\ 1_{\mathrm{V}}\ 1_{\mathrm{V'}} & \longrightarrow & \text{Eq. (101)} \\ \xrightarrow{1} & 5_{\mathrm{F}}\ 1_{\mathrm{V}}\ 1_{\mathrm{V'}} & \longrightarrow & \text{Eq. (102)} \\ \xrightarrow{1} & 5_{\mathrm{E}}\ 1_{\mathrm{V}}\ 1_{\mathrm{V'}} & \xrightarrow{2',2} & \mathrm{G_{SM}}\ Z_2 \\ \xrightarrow{2} & 5\ 1_{\mathrm{V'}}\ Z_2 & \longrightarrow & \cdots \\ \xrightarrow{1,2} & 5\ 1_{\mathrm{V}} & \longrightarrow & \cdots \\ \xrightarrow{1} & 5_{\mathrm{F}}\ 1_{\mathrm{V}} & \xrightarrow{2',2} & \mathrm{G_{SM}}\ Z_2 \\ \xrightarrow{1} & \mathrm{G_{SM}}\ 1_{\mathrm{V}} & \xrightarrow{2} & \mathrm{G_{SM}}\ Z_2 \\ \xrightarrow{1,2} & \mathrm{G_{SM}}\ 1_{\mathrm{V'}}\ Z_2 & \xrightarrow{2} & \mathrm{G_{SM}}\ Z_2 \\ \xrightarrow{1,2} & 4_{\mathrm{C}}\ 2_{\mathrm{L}}\ 2_{\mathrm{R}}\ 1_{\mathrm{V'}} & \longrightarrow & \text{Eq. (103)} \\ \xrightarrow{1} & 4_{\mathrm{C}}\ 2_{\mathrm{L}}\ 2_{\mathrm{R}} & \longrightarrow & \text{Eq. (97)} \\ \xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 2_{\mathrm{R}}\ 1_{\mathrm{B-L}}\ 1_{\mathrm{V'}} & \longrightarrow & \cdots \\ \xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{\mathrm{R}}\ 1_{\mathrm{B-L}}\ 1_{\mathrm{V'}} & \longrightarrow & \cdots \end{array} \right. \tag{99}$$

with more direct schemes

$$\mathrm{E}_6 \left\{ \begin{array}{llll} \xrightarrow{1} & 5\ 1_{\mathrm{V}}\ 1_{\mathrm{V'}} & \longrightarrow & \text{Eq.(101)} \\ \xrightarrow{1} & 5_{\mathrm{F}}\ 1_{\mathrm{V}}\ 1_{\mathrm{V'}} & \longrightarrow & \text{Eq.(102)} \\ \xrightarrow{1} & 5_{\mathrm{E}}\ 1_{\mathrm{V}}\ 1_{\mathrm{V'}} & \xrightarrow{2',2} & \mathrm{G_{SM}}\ Z_2 \\ \xrightarrow{1} & 5\ 1_{\mathrm{V}} & \longrightarrow & \cdots \\ \xrightarrow{1} & 5\ 1_{\mathrm{V'}} & \longrightarrow & \cdots \\ \xrightarrow{1} & 5_{\mathrm{F}}\ 1_{\mathrm{V}} & \xrightarrow{2',2} & \mathrm{G_{SM}}\ Z_2 \\ \xrightarrow{1} & 4_{\mathrm{C}}\ 2_{\mathrm{L}}\ 2_{\mathrm{R}}\ 1_{\mathrm{V'}} & \longrightarrow & \text{Eq. (103)} \\ \xrightarrow{1} & 4_{\mathrm{C}}\ 2_{\mathrm{L}}\ 2_{\mathrm{R}} & \longrightarrow & \text{Eq. (97)} \\ \xrightarrow{1} & 4_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{\mathrm{R}} & \longrightarrow & \cdots \\ \xrightarrow{1} & 4_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{\mathrm{R}}\ 1_{\mathrm{V'}} & \longrightarrow & \cdots \\ \xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 2_{\mathrm{R}}\ 1_{\mathrm{B-L}}\ 1_{\mathrm{V'}} & \longrightarrow & \cdots \\ \xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{\mathrm{R}}\ 1_{\mathrm{B-L}}\ 1_{\mathrm{V'}} & \longrightarrow & \cdots \\ \xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{\mathrm{R}}\ 1_{\mathrm{B-L}} & \xrightarrow{2} & \mathrm{G_{SM}}\ Z_2 \\ \xrightarrow{1} & \mathrm{G_{SM}}\ 1_{\mathrm{V}} & \xrightarrow{2} & \mathrm{G_{SM}}\ Z_2 \\ \xrightarrow{1,2} & \mathrm{G_{SM}}\ 1_{\mathrm{V'}}\ Z_2 & \xrightarrow{2} & \mathrm{G_{SM}}\ Z_2 \end{array} \right. \tag{100}$$

where

$$5\ 1_{\mathrm{V}}\ 1_{\mathrm{V'}} \left\{ \begin{array}{lll} \xrightarrow{2} & 5\ 1_{\mathrm{V'}}\ Z_2 & \left\{ \xrightarrow{1} \ \mathrm{G_{SM}}\ 1_{\mathrm{V'}}\ Z_2 \xrightarrow{2} \mathrm{G_{SM}}\ Z_2 \right. \\ \xrightarrow{1} & \mathrm{G_{SM}}\ 1_{\mathrm{V}}\ 1_{\mathrm{V'}} & \left\{ \begin{array}{llll} \xrightarrow{2} & \mathrm{G_{SM}}\ 1_{\mathrm{V}} & \xrightarrow{2} & \mathrm{G_{SM}}\ Z_2 \\ \xrightarrow{2} & \mathrm{G_{SM}}\ 1_{\mathrm{V'}}\ Z_2 & \xrightarrow{2} & \mathrm{G_{SM}}\ Z_2 \end{array} \right. \\ \xrightarrow{2} & 5\ 1_{\mathrm{V}} & \longrightarrow \ \cdots \end{array} \right. \tag{101}$$

$$5_{\mathrm{F}}\ 1_{\mathrm{V}}\ 1_{\mathrm{V'}} \left\{ \begin{array}{llll} \xrightarrow{2} & 5_{\mathrm{F}}\ 1_{\mathrm{V}} & \xrightarrow{2',2} & \mathrm{G_{SM}}\ Z_2 \\ \xrightarrow{2',2} & \mathrm{G_{SM}}\ Z_2 & & \end{array} \right. \tag{102}$$

$$4_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm V'} \left\{ \begin{array}{lll}
\xrightarrow{2} & 4_{\rm C}\ 2_{\rm L}\ 2_{\rm R} & \longrightarrow \quad \text{Eq. (97)} \\
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L}\ 1_{\rm V'} & \left\{ \begin{array}{llll}
\xrightarrow{2} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} & \xrightarrow{2} & {\rm G_{SM}}\ Z_2 \\
\xrightarrow{2',2} & {\rm G_{SM}} 1_{\rm V'}\ Z_2 & \xrightarrow{2} & {\rm G_{SM}}\ Z_2 \\
\xrightarrow{2',2} & {\rm G_{SM}}\ Z_2 & &
\end{array} \right. \\
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L}\ 1_{\rm V'} & \left\{ \begin{array}{llll}
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L}\ 1_{\rm V'} & \longrightarrow & \cdots \\
\xrightarrow{2} & 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L} & \longrightarrow & \cdots \\
\xrightarrow{2',2} & {\rm G_{SM}} 1_{\rm V'}\ Z_2 & \xrightarrow{2} & {\rm G_{SM}}\ Z_2 \\
\xrightarrow{1,2} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} & \xrightarrow{2} & {\rm G_{SM}}\ Z_2 \\
\xrightarrow{2',2} & {\rm G_{SM}}\ Z_2 & &
\end{array} \right. \\
\xrightarrow{1} & 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm V'} & \left\{ \begin{array}{llll}
\xrightarrow{2} & 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R} & \longrightarrow & \cdots \\
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L}\ 1_{\rm V'} & \longrightarrow & \cdots \\
\xrightarrow{2',2} & {\rm G_{SM}} 1_{\rm V'}\ Z_2 & \xrightarrow{2} & {\rm G_{SM}}\ Z_2 \\
\xrightarrow{1,2} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} & \xrightarrow{2} & {\rm G_{SM}}\ Z_2 \\
\xrightarrow{2} & {\rm G_{SM}}\ Z_2 & &
\end{array} \right. \\
\xrightarrow{1,2} & {\rm G_{SM}} 1_{\rm V'}\ Z_2 & \xrightarrow{2} \quad {\rm G_{SM}}\ Z_2 \\
\xrightarrow{1,2} & 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L} & \longrightarrow \quad \cdots \\
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} & \xrightarrow{2} \quad {\rm G_{SM}}\ Z_2
\end{array} \right. \tag{103}$$

We proceed with the allowed SSB patterns of E_6 down to the $\mathrm{G_{SM}} \times Z_2$ via $\mathrm{SU(3)_C} \times \mathrm{SU(3)_L} \times \mathrm{SU(3)_R}$.

$$\mathrm{E}_6 \xrightarrow{0} 3_{\rm C}\, 3_{\rm L}\, 3_{\rm R} \left\{ \begin{array}{llll}
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 2_{\rm (R)}\ 1_{\rm Y_{(R)}}\ 1_{\rm Y_L} & \longrightarrow & \text{Eq. (106)} \\
\xrightarrow{1} & 3_{\rm C}\ 3_{\rm L}\ 2_{\rm (R)} 1_{\rm (Y_R)} & \longrightarrow & \text{Eq. (107)} \\
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 3_{\rm R}\ 1_{\rm Y_L} & \longrightarrow & \text{Eq. (108)} \\
\xrightarrow{1} & 3_{\rm C}\ 3_{\rm L}\ 1_{\rm (R)}\ 1_{\rm Y_{(R)}} & \longrightarrow & \text{Eq. (109)} \\
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm (R)}\ 1_{\rm Y_{(R)}}\ 1_{\rm Y_L} & \longrightarrow & \cdots \\
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 2_{\rm (R)}\ 1_{\rm B-L} & \longrightarrow & \cdots \\
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm (R)}\ 1_{\rm B-L} & \xrightarrow{2} & {\rm G_{SM}}\ Z_2
\end{array} \right. \tag{104}$$

with more direct breakings

$$\mathrm{E}_6 \left\{ \begin{array}{llll}
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 2_{\rm (R)}\ 1_{\rm Y_L}\ 1_{\rm Y_{(R)}} & \longrightarrow & \text{Eq. (106)} \\
\xrightarrow{1} & 3_{\rm C}\ 3_{\rm L}\ 2_{\rm (R)}\ 1_{\rm Y_{(R)}} & \longrightarrow & \text{Eq. (107)} \\
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 3_{\rm R}\ 1_{\rm Y_L} & \longrightarrow & \text{Eq. (108)} \\
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm (R)}\ 1_{\rm Y_L}\ 1_{\rm Y_{(R)}} & \longrightarrow & \cdots \\
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 2_{\rm (R)}\ 1_{\rm B-L} & \longrightarrow & \cdots \\
\xrightarrow{1} & 3_{\rm C}\ 2_{\rm L}\ 1_{\rm (R)}\ 1_{\rm B-L} & \xrightarrow{2} & {\rm G_{SM}}\ Z_2
\end{array} \right. \tag{105}$$

where

$$
3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 2_{(\mathrm{R})}\ 1_{\mathrm{Y_L}}\ 1_{\mathrm{Y_{(R)}}}
\left\{
\begin{array}{llll}
\xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 2_{(\mathrm{R})}\ 1_{\mathrm{B-L}} & \left\{ \begin{array}{lll} \xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{(\mathrm{R})}\ 1_{\mathrm{B-L}} & \xrightarrow{2}\ \mathrm{G_{SM}}\ Z_2 \\ \xrightarrow{2',2} & \mathrm{G_{SM}}\ Z_2 & \end{array} \right. \\
\xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{(\mathrm{R})}\ 1_{\mathrm{Y_{(R)}}}\ 1_{\mathrm{Y_L}} & \left\{ \begin{array}{lll} \xrightarrow{2} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{(\mathrm{R})}\ 1_{\mathrm{B-L}} & \xrightarrow{2}\ \mathrm{G_{SM}}\ Z_2 \\ \xrightarrow{2} & \mathrm{G_{SM}}\ Z_2 & \end{array} \right. \\
\xrightarrow{1,2} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{(\mathrm{R})}\ 1_{\mathrm{B-L}} & \xrightarrow{2}\ \mathrm{G_{SM}}\ Z_2 \\
\xrightarrow{2',2} & \mathrm{G_{SM}}\ Z_2 &
\end{array}
\right.
\qquad (106)
$$

$$
3_{\mathrm{C}}\ 3_{\mathrm{L}}\ 2_{(\mathrm{R})} 1_{(\mathrm{Y_R})}
\left\{
\begin{array}{llll}
\xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 2_{(\mathrm{R})}\ 1_{\mathrm{Y_{(R)}}}\ 1_{\mathrm{Y_L}} & \longrightarrow & \text{Eq. (106)} \\
\xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{(\mathrm{R})}\ 1_{\mathrm{Y_{(R)}}}\ 1_{\mathrm{Y_L}} & \longrightarrow & \cdots \\
\xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 2_{(\mathrm{R})}\ 1_{\mathrm{B-L}} & \longrightarrow & \cdots \\
\xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{(\mathrm{R})}\ 1_{\mathrm{B-L}} & \xrightarrow{2} & \mathrm{G_{SM}}\ Z_2 \\
\xrightarrow{2',2} & \mathrm{G_{SM}}\ Z_2 & &
\end{array}
\right.
\qquad (107)
$$

$$
3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 3_{\mathrm{R}}\ 1_{\mathrm{Y_L}}
\left\{
\begin{array}{llll}
\xrightarrow{1} & 3_{\mathrm{C}}\ 2_{(\mathrm{R})}\ 2_{\mathrm{L}}\ 1_{\mathrm{Y_L}}\ 1_{\mathrm{Y_{(R)}}} & \longrightarrow & \text{Eq. (106)} \\
\xrightarrow{2'} & 3_{\mathrm{C}}\ 2_{(\mathrm{R})}\ 2_{\mathrm{L}}\ 1_{\mathrm{B-L}} & \longrightarrow & \cdots \\
\xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{(\mathrm{R})}\ 1_{\mathrm{Y_L}}\ 1_{\mathrm{Y_{(R)}}} & \longrightarrow & \cdots \\
\xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{(\mathrm{R})}\ 1_{\mathrm{B-L}} & \xrightarrow{2} & \mathrm{G_{SM}}\ Z_2 \\
\xrightarrow{2',2} & \mathrm{G_{SM}}\ Z_2 & &
\end{array}
\right.
\qquad (108)
$$

and

$$
3_{\mathrm{C}}\ 3_{\mathrm{L}}\ 1_{(\mathrm{R})}\ 1_{\mathrm{Y_{(R)}}}
\left\{
\begin{array}{llll}
\xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{\mathrm{Y_L}}\ 1_{(\mathrm{R})}\ 1_{\mathrm{Y_{(R)}}} & \longrightarrow & \cdots \\
\xrightarrow{2'} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{(\mathrm{R})}\ 1_{\mathrm{B-L}} & \xrightarrow{2} & \mathrm{G_{SM}}\ Z_2 \\
\xrightarrow{2',2} & \mathrm{G_{SM}}\ Z_2 & &
\end{array}
\right.
\qquad (109)
$$

Finally, we list below the SSB schemes of E_6 down to the $\mathrm{G_{SM}} \times Z_2$ via $\mathrm{SU}(6) \times \mathrm{SU}(2)$. These are:

$$
\begin{array}{ccc}
\mathrm{E}_6 & \xrightarrow{0} & 6\ 2_{\mathrm{L}} \\
& \text{or} & \mathrm{E}_6
\end{array}
\left\{
\begin{array}{llll}
\xrightarrow{1} & 3_{\mathrm{C}}\ 3_{\mathrm{R}}\ 2_{\mathrm{L}}\ 1_{\mathrm{Y_L}} & \longrightarrow & \text{Eq. (108)} \\
\xrightarrow{1} & 4_{\mathrm{C}}\ 2_{\mathrm{L}}\ 2_{\mathrm{R}}\ 1_{\mathrm{V'}} & \longrightarrow & \text{Eq. (103)} \\
\xrightarrow{0} & 4_{\mathrm{C}}\ 2_{\mathrm{L}}\ 2_{\mathrm{R}} & \longrightarrow & \cdots \\
\xrightarrow{1} & 4_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{\mathrm{R}}\ 1_{\mathrm{V'}} & \longrightarrow & \cdots \\
\xrightarrow{1} & 4_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{\mathrm{R}} & \longrightarrow & \cdots \\
\xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 2_{(\mathrm{R})}\ 1_{\mathrm{B-L}} & \longrightarrow & \cdots \\
\xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{(\mathrm{R})}\ 1_{\mathrm{Y_L}}\ 1_{\mathrm{Y_{(R)}}} & \longrightarrow & \cdots \\
\xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{(\mathrm{R})}\ 1_{\mathrm{B-L}} & \xrightarrow{2} & \mathrm{G_{SM}}\ Z_2
\end{array}
\right.
\qquad (110)
$$

and

$$
\begin{array}{ccc}
\mathrm{E}_6 & \xrightarrow{0} & 6\ 2_{\mathrm{R}} \\
& \text{or} & \mathrm{E}_6
\end{array}
\left\{
\begin{array}{llll}
\xrightarrow{1} & 4_{\mathrm{C}}\ 2_{\mathrm{L}}\ 2_{\mathrm{R}}\ 1_{V'} & \longrightarrow & \text{Eq. (103)} \\
\xrightarrow{1} & 4_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{\mathrm{R}}\ 1_{\mathrm{V'}} & \longrightarrow & \cdots \\
\xrightarrow{0} & 4_{\mathrm{C}}\ 2_{\mathrm{L}}\ 2_{\mathrm{R}} & \longrightarrow & \cdots \\
\xrightarrow{1} & 4_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{\mathrm{R}} & \longrightarrow & \cdots \\
\xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 2_{\mathrm{R}}\ 1_{\mathrm{B-L}}\ 1_{\mathrm{V'}} & \longrightarrow & \cdots \\
\xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 2_{\mathrm{R}}\ 1_{\mathrm{B-L}} & \longrightarrow & \cdots \\
\xrightarrow{1} & 3_{\mathrm{C}}\ 2_{\mathrm{L}}\ 1_{\mathrm{R}}\ 1_{\mathrm{B-L}} & \xrightarrow{2} & \mathrm{G_{SM}}\ Z_2
\end{array}
\right.
\qquad (111)
$$

As it was shown in Ref. [5], the SSB schemes of SU(6) and SU(7) down to the standard model which could accommodate an inflationary era with no defect (of any kind) at later times are inconsistent with proton lifetime measurements and minimal SU(6) and SU(7) does not predict neutrino masses. Thus, these models are incompatible with high energy physics phenomenology.